Section 4.4 Page 254 Question 8

Let x represent the width of the corral. Then, $30 - 2x$ represents the length. Solve $x(30 - 2x) = 100$ to find the dimensions.

$$\begin{aligned} x(30 - 2x) &= 100 \\ -2x^2 + 30x - 100 &= 0 \\ x^2 - 15x + 50 &= 0 \\ (x - 10)(x + 5) &= 0 \end{aligned}$$

$x - 10 = 0$ or $x + 5 = 0$

$x = 10$ $\quad$ $x = -5$

Since the width cannot be negative, $x = -5$ is an extraneous root.

The dimensions of the corral are 10 m by 10 m.

Section 4.4 Page 255 Question 9

Let x represent the width of the border. Then, the dimensions of the mural are $15 - 2x$ by $12 - 2x$ with an area of 135 m^2. Solve $(15 - 2x)(12 - 2x) = 135$ to find the width of the border.

$(15 - 2x)(12 - 2x) = 135$

$4x^2 - 54x + 180 = 135$

$4x^2 - 54x + 45 = 0$

Substitute into the quadratic formula, $a = 4$, $b = -54$, $c = 45$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-54) \pm \sqrt{(-54)^2 - 4(4)(45)}}{2(4)}$$

$$x = \frac{54 \pm \sqrt{2916 - 720}}{8}$$

$$x = \frac{54 \pm \sqrt{2196}}{8}$$

$$x = \frac{54 + \sqrt{2196}}{8} \quad \text{or} \quad x = \frac{54 - \sqrt{2196}}{8}$$

$x \approx 12.61$ $\quad$ $x \approx 0.89$

Since the border cannot be wider than one of the dimensions, $x = 12.61$ is an extraneous root.

The width of the border is 0.89 m, to the nearest hundredth of a metre.

Section 4.4 Page 255 Question 10

Let x represent the number. Solve $\frac{1}{2}x^2 - x = 11$ to find the number.

$$\frac{1}{2}x^2 - x - 11 = 0$$

Substitute into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4\left(\frac{1}{2}\right)(-11)}}{2\left(\frac{1}{2}\right)}$$

$$x = \frac{1 \pm \sqrt{1 + 22}}{1}$$

$$x = 1 \pm \sqrt{23}$$

$x = 1 + \sqrt{23}$ or $x = 1 - \sqrt{23}$

$x \approx 5.80$ $\quad$ $x \approx -3.80$

The exact number is $1 + \sqrt{23}$ or $1 - \sqrt{23}$. The number, to the nearest hundredth, is 5.80 or –3.80.

Section 4.4 Page 255 Question 11

Solve $0 = -0.4(d - 2.5)^2 + 2.5$ to find the width of the arch.

$0 = -0.4(d - 2.5)^2 + 2.5$

$0 = -0.4d^2 + 2d$

$0 = d^2 - 5d$

$0 = d(d - 5)$

$d = 0$ or $d - 5 = 0$

$d = 5$

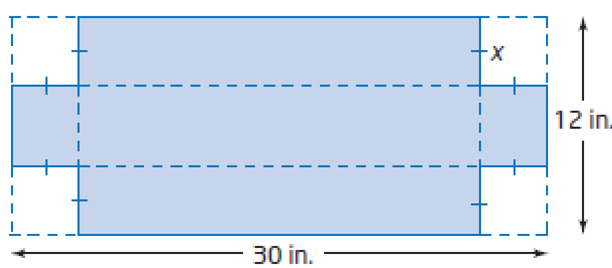
The arch is 5 – 0, or 5 m wide.

Section 4.4 Page 255 Question 12

a) $SA_{base} = (12 - 2x)(30 - 2x)$

$208 = 4x^2 - 84x + 360$

$0 = 4x^2 - 84x + 152$



b) $0 = 4x^2 - 84x + 152$
$0 = x^2 - 21x + 38$
$0 = (x - 19)(x - 2)$
$x - 19 = 0$ or $x - 2 = 0$
$x = 19$ $x = 2$
Since the side length of the corner square cannot be greater than a dimension of the cardboard, $x = 19$ is an extraneous root. The side length of the square cut from each corner is 2 in.

c) The dimensions of the box are 26 in. by 8 in. by 2 in.

Section 4.4 Page 255 Question 13

a) $42 = 0.0067v^2 + 0.15v$
$0 = 0.0067v^2 + 0.15v - 42$
Substitute into the quadratic formula.

$$v = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$v = \frac{-0.15 \pm \sqrt{0.15^2 - 4(0.0067)(-42)}}{2(0.0067)}$$

$$v = \frac{-0.15 \pm \sqrt{0.0225 + 1.1256}}{0.0134}$$

$$v = \frac{-0.15 \pm \sqrt{1.1481}}{0.0134}$$

$$v = \frac{-0.15 + \sqrt{1.1481}}{0.0134} \quad \text{or} \quad v = \frac{-0.15 - \sqrt{1.1481}}{0.0134}$$

$v \approx 68.8$ $v \approx -91.2$
Since speed cannot be negative, $x = -91.2$ is an extraneous root.
The car can be travelling at approximately 68.8 km/h to be able to stop in 42 m.

b) $75 = 0.0067v^2 + 0.15v$
$0 = 0.0067v^2 + 0.15v - 75$
Substitute into the quadratic formula.

$$v = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$v = \frac{-0.15 \pm \sqrt{0.15^2 - 4(0.0067)(-75)}}{2(0.0067)}$$
$$v = \frac{-0.15 \pm \sqrt{0.0225 + 2.01}}{0.0134}$$
$$v = \frac{-0.15 \pm \sqrt{2.0325}}{0.0134}$$
$$v = \frac{-0.15 + \sqrt{2.0325}}{0.0134} \quad \text{or} \quad v = \frac{-0.15 - \sqrt{2.0325}}{0.0134}$$
$$v \approx 95.2 \qquad\qquad v \approx -117.6$$
Since speed cannot be negative, $x = -117.6$ is an extraneous root.
The car can be travelling at approximately 95.2 km/h to be able to stop in 75 m.

c) $135 = 0.0067v^2 + 0.15v$
$0 = 0.0067v^2 + 0.15v - 135$
Substitute into the quadratic formula.
$$v = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$v = \frac{-0.15 \pm \sqrt{0.15^2 - 4(0.0067)(-135)}}{2(0.0067)}$$
$$v = \frac{-0.15 \pm \sqrt{0.0225 + 3.618}}{0.0134}$$
$$v = \frac{-0.15 \pm \sqrt{3.6405}}{0.0134}$$
$$v = \frac{-0.15 + \sqrt{3.6405}}{0.0134} \quad \text{or} \quad v = \frac{-0.15 - \sqrt{3.6405}}{0.0134}$$
$$v \approx 131.2 \qquad\qquad v \approx -153.6$$
Since speed cannot be negative, $x = -153.6$ is an extraneous root.
The car can be travelling at approximately 131.2 km/h to be able to stop in 135 m.

Section 4.4 Page 255 Question 14

a) $A(t) = 0.3t^2 + 0.1t + 4.2$
$A(0) = 0.3(0)^2 + 0.1(0) + 4.2$
$A(0) = 4.2$
At $t = 0$, the level of carbon dioxide in the air is 4.2 ppm.

b) Solve $8 = 0.3t^2 + 0.1t + 4.2$ using the quadratic formula.

$8 = 0.3t^2 + 0.1t + 4.2$

$0 = 0.3t^2 + 0.1t - 3.8$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-0.1 \pm \sqrt{0.1^2 - 4(0.3)(-3.8)}}{2(0.3)}$$

$$t = \frac{-0.1 \pm \sqrt{4.57}}{0.6}$$

$$t = \frac{-0.1 + \sqrt{4.57}}{0.6} \quad \text{or} \quad t = \frac{-0.1 - \sqrt{4.57}}{0.6}$$

$t \approx 3.4 \qquad t \approx -3.7$

Since time cannot be negative, $x = -3.7$ is an extraneous root.

In 3.4 years, to the nearest tenth of a year, the carbon monoxide level will be 8 ppm.

Section 4.4 Page 256 Question 15

Let n represent the number of price decreases. The new price is $275 - 15n$.
The new number of ski jackets sold is $90 + 5n$.
The revenue is \$19 600.
Revenue = (price)(number of sessions)

$19\,600 = (275 - 15n)(90 + 5n)$

$19\,600 = -75n^2 + 25n + 24\,750$

$0 = -75n^2 + 25n + 5150$

$0 = 3n^2 - n - 206$

Substitute into the quadratic formula.

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$n = \frac{-1 \pm \sqrt{1^2 - 4(3)(-206)}}{2(3)}$$

$$n = \frac{-1 \pm \sqrt{2473}}{6}$$

$$n = \frac{-1 + \sqrt{2473}}{6} \quad \text{or} \quad n = \frac{-1 - \sqrt{2473}}{6}$$

$n \approx 8 \qquad n \approx -8$

Since the number of price decreases must be positive, $x = -8$ is an extraneous root.

The lowest price that would produce revenues of at least \$19 600 is 275 – 15(8), or \$155.

At this price, 90 + 5(8), or 130 jackets would be sold.

Section 4.4 Page 256 Question 21

In Line 1, the wrong value for b was substituted outside the radical.
In Line 2, the expression $-4(-3)(2)$ was incorrectly evaluated.
The corrected solution is as follows.
For $-3x^2 - 7x + 2 = 0$, $a = -3$, $b = -7$, and $c = 2$.

Line 1: $x = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4(-3)(2)}}{2(-3)}$

Line 2: $x = \dfrac{7 \pm \sqrt{49 + 24}}{-6}$

Line 3: $x = \dfrac{7 \pm \sqrt{73}}{-6}$

Line 4: $x = \dfrac{-7 \pm \sqrt{73}}{6}$

Line 5: So, $x = \dfrac{-7 + \sqrt{73}}{6}$ or $x = \dfrac{-7 - \sqrt{73}}{6}$.

Section 4.4 Page 256 Question 22

a) The roots of a quadratic equation are the same as the x-intercepts of the graph of the corresponding quadratic function. So, the x-intercepts are $x = \dfrac{3 \pm \sqrt{25}}{2}$, or $x = 4$ and $x = -1$.

b) The axis of symmetry is halfway between the two roots of -1 and 4. So, the equation of the axis of symmetry is $x = 2$.

Section 4.4 Page 257 Question 23

Example: If the quadratic is easily factored, then factoring is faster. If it cannot be factored, then completing the square or applying the quadratic formula are other ways to determine exact answers. Graphing with technology is a quick way of finding out if there are real solutions. However, the roots found may be approximate.