Chapter 8 Systems of Equations

Section 8.1 Solving Systems of Equations Graphically

Section 8.1 Page 435 Question 1

a) System A models the situation: to go off a ramp at different heights means two positive vertical intercepts and in this system the launch angles are different causing the bike with the lower trajectory to land sooner. System B is not correct because it shows both jumps starting from same height. System C has one start from zero, which would mean no ramp. In System D, a steeper trajectory would mean being in the air longer but the rider is going at the same speed.

b) The rider was at the same height and at same time after leaving the jump regardless of which ramp was chosen.

Section 8.1 Page 435 Question 2

For $(0, -5)$:
In $y = -x^2 + 4x - 5$

Left Side	Right Side
$y = -5$	$-x^2 + 4x - 5$
	$= -(0)^2 + 4(0) - 5$
	$= -5$

Left Side = Right Side

In $y = x - 5$

Left Side	Right Side
$y = -5$	$x - 5$
	$= 0 - 5$
	$= -5$

Left Side = Right Side

For $(3, -2)$:
In $y = -x^2 + 4x - 5$

Left Side	Right Side
$y = -2$	$-x^2 + 4x - 5$
	$= -(3)^2 + 4(3) - 5$
	$= -2$

Left Side = Right Side

In $y = x - 5$

Left Side	Right Side
$y = -2$	$x - 5$
	$= 3 - 5$
	$= -2$

Left Side = Right Side

So, both solutions are verified.

Section 8.1 Page 435 Question 3

a) The equations $x + y + 3 = 0$ and $x^2 + 6x + y + 7 = 0$ define a linear-quadratic system. The solutions are $(-4, 1)$ and $(-1, -2)$.

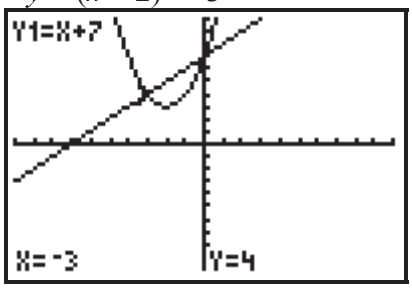
b) The equations $y = x^2 - 4x + 7$ and $y = \frac{1}{2}x^2 - 2x + 3$ define a quadratic-quadratic system. The system has no solution.

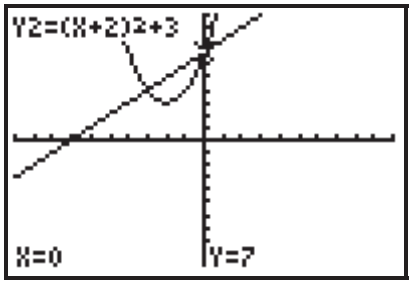
c) The equations $y = 2x^2 - 4x - 2$ and $y = -4$ define a linear-quadratic system. The solution is $(1, -4)$.

Section 8.1 Page 435 Question 4

a) $y = x + 7$

$y = (x + 2)^2 + 3$





From the graph, the solutions are $(-3, 4)$ and $(0, 7)$.

For $(-3, 4)$:

In $y = x + 7$

Left Side	Right Side
$y = 4$	$x + 7$
	$= -3 + 7$
	$= 4$

Left Side = Right Side

In $y = (x + 2)^2 + 3$

Left Side	Right Side
$y = 4$	$(x + 2)^2 + 3$
	$= (-3 + 2)^2 + 3$
	$= 1 + 3$
	$= 4$

Left Side = Right Side

For (0, 7):

In $y = x + 7$

Left Side	Right Side
$y = 7$	$x + 7$
	$= 0 + 7$
	$= 7$

Left Side = Right Side

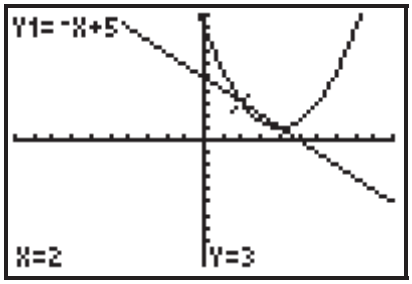
In $y = (x + 2)^2 + 3$

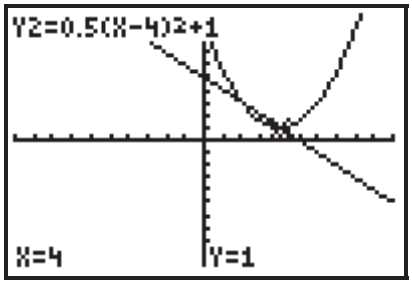
Left Side	Right Side
$y = 7$	$(x + 2)^2 + 3$
	$= (0 + 2)^2 + 3$
	$= 4 + 3$
	$= 7$

Left Side = Right Side

b) $f(x) = -x + 5$

$g(x) = \frac{1}{2}(x-4)^2 + 1$





From the graph, the solutions are (2, 3) and (4, 1).

For (2, 3):

In $f(x) = -x + 5$

Left Side	Right Side
$f(x) = 3$	$-x + 5$
	$= -2 + 5$
	$= 3$

Left Side = Right Side

In $g(x) = \frac{1}{2}(x-4)^2 + 1$

Left Side	Right Side
$g(x) = 3$	$\frac{1}{2}(x-4)^2 + 1$
	$= \frac{1}{2}(2-4)^2 + 1$
	$= 3$

Left Side = Right Side

For (4, 1):

In $f(x) = -x + 5$

Left Side	Right Side
$f(x) = 1$	$-x + 5$
	$= -4 + 5$
	$= 1$

Left Side = Right Side

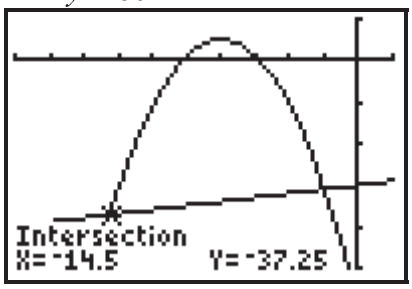
In $g(x) = \frac{1}{2}(x-4)^2 + 1$

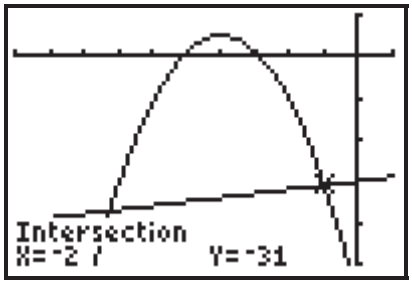
Left Side	Right Side
$g(x) = 1$	$\frac{1}{2}(x-4)^2 + 1$
	$= \frac{1}{2}(4-4)^2 + 1$
	$= 1$

Left Side = Right Side

c) $x^2 + 16x + y = -59$

$x - 2y = 60$





From the graph, the solutions are (–14.5, –37.25) and (–2, –31).

For (–14.5, –37.25):
In $x^2 + 16x + y = -59$

Left Side	Right Side
$x^2 + 16x + y$	-59
$= (-14.5)^2 + 16(-14.5) + (-37.25)$	
$= 210.25 - 232 - 37.25$	
$= -59$	

Left Side = Right Side

In $x - 2y = 60$

Left Side	Right Side
$x - 2y$	60
$= -14.5 - 2(-37.25)$	
$= 60$	

Left Side = Right Side

For (–2, –31):
In $x^2 + 16x + y = -59$

Left Side	Right Side
$x^2 + 16x + y$	-59
$= (-2)^2 + 16(-2) + (-31)$	
$= 4 - 32 - 31$	
$= -59$	

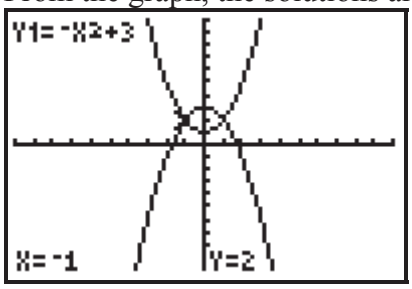
Left Side = Right Side

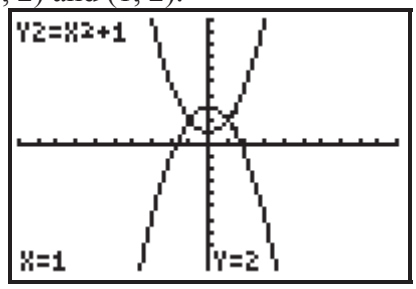
In $x - 2y = 60$

Left Side	Right Side
$x - 2y$	60
$= -2 - 2(-31)$	
$= 60$	

Left Side = Right Side

d) $x^2 + y - 3 = 0$
$x^2 - y + 1 = 0$
From the graph, the solutions are (–1, 2) and (1, 2).





For (–1, 2):
In $x^2 + y - 3 = 0$

Left Side	Right Side
$x^2 + y - 3$	0
$= (-1)^2 + 2 - 3$	
$= 0$	

Left Side = Right Side

In $x^2 - y + 1 = 0$

Left Side	Right Side
$x^2 - y + 1$	0
$= (-1)^2 - 2 + 1$	
$= 0$	

Left Side = Right Side

For (1, 2):
In $x^2 + y - 3 = 0$

Left Side	Right Side
$x^2 + y - 3$	$= 0$
$= (1)^2 + 2 - 3$	
$= 0$	

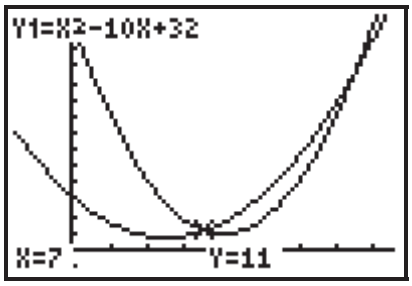
Left Side = Right Side

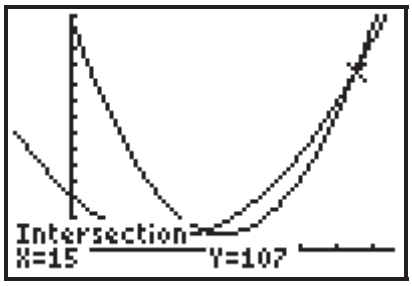
In $x^2 - y + 1 = 0$

Left Side	Right Side
$x^2 - y + 1$	0
$= (1)^2 - 2 + 1$	
$= 0$	

Left Side = Right Side

e) $y = x^2 - 10x + 32$
$y = 2x^2 - 32x + 137$





From the graph, the solutions are (7, 11) and (15, 107).

For (7, 11):

In $y = x^2 - 10x + 32$

Left Side	Right Side
y	$x^2 - 10x + 32$
$= 11$	$= 7^2 - 10(7) + 32$
	$= 49 - 70 + 32$
	$= 11$

Left Side = Right Side

In $y = 2x^2 - 32x + 137$

Left Side	Right Side
y	$x^2 - 10x + 32$
$= 11$	$= 2(7)^2 - 32(7) + 137$
	$= 98 - 224 + 137$
	$= 11$

Left Side = Right Side

For (15, 107):

In $y = x^2 - 10x + 32$

Left Side	Right Side
y	$x^2 - 10x + 32$
$= 107$	$= 15^2 - 10(15) + 32$
	$= 225 - 150 + 32$
	$= 107$

Left Side = Right Side

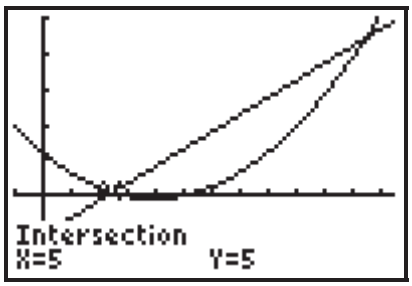
In $y = 2x^2 - 32x + 137$

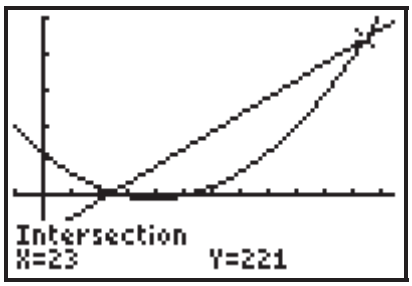
Left Side	Right Side
y	$2x^2 - 32x + 137$
$= 107$	$= 2(15)^2 - 32(15) + 137$
	$= 450 - 480 + 137$
	$= 107$

Left Side = Right Side

Section 8.1 Page 436 Question 5

a) $h = d^2 - 16d + 60$
$h = 12d - 55$





From the graph, the solutions are (5, 5) and (23, 221).

For (5, 5):
In $h = d^2 - 16d + 60$

Left Side	Right Side
h	$d^2 - 16d + 60$
$= 5$	$= 5^2 - 16(5) + 60$
	$= 25 - 80 + 60$
	$= 5$

Left Side = Right Side

In $h = 12d - 55$

Left Side	Right Side
h	$12d - 55$
$= 5$	$= 12(5) - 55$
	$= 5$

Left Side = Right Side

For (23, 221):
In $h = d^2 - 16d + 60$

Left Side	Right Side
h	$d^2 - 16d + 60$
$= 221$	$= 23^2 - 16(23) + 60$
	$= 529 - 368 + 60$
	$= 221$

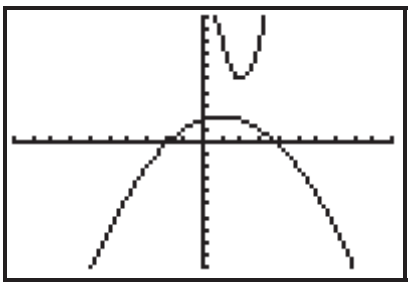
Left Side = Right Side

In $h = 12d - 55$

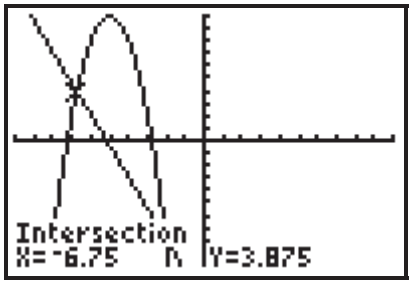
Left Side	Right Side
h	$12d - 55$
$= 221$	$= 12(23) - 55$
	$= 221$

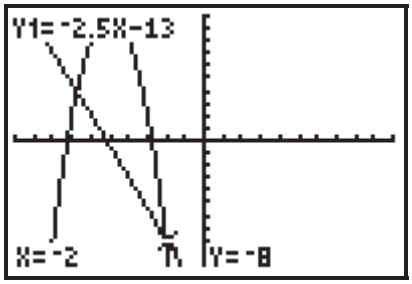
Left Side = Right Side

b) $p = 3q^2 - 12q + 17$
$p = -0.25q^2 + 0.5q + 1.75$
From the graph, there are no solutions.



c) $2v^2 + 20v + t = -40$
$5v + 2t + 26 = 0$





From the graph, the solutions are (–6.75, 3.875) and (–2, –8).

For (–6.75, 3.875):
In $2v^2 + 20v + t = -40$

Left Side	Right Side
$2v^2 + 20v + t$	–40
$= 2(-6.75)^2 + 20(-6.75) + 3.875$	
$= 91.125 - 135 + 3.875$	
$= -40$	

Left Side = Right Side

In $5v + 2t + 26 = 0$

Left Side	Right Side
$5v + 2t + 26$	0
$= 5(-6.75) + 2(3.875) + 26$	
$= 0$	

Left Side = Right Side

For (–2, –8):
In $2v^2 + 20v + t = -40$

Left Side	Right Side
$2v^2 + 20v + t$	–40
$= 2(-2)^2 + 20(-2) + (-8)$	
$= 8 - 40 + 8$	
$= -40$	

Left Side = Right Side

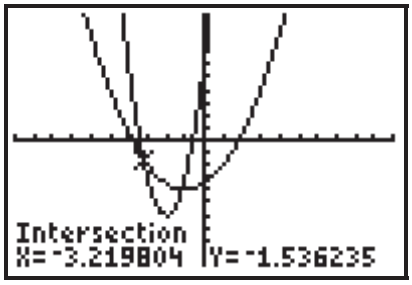
In $5v + 2t + 26 = 0$

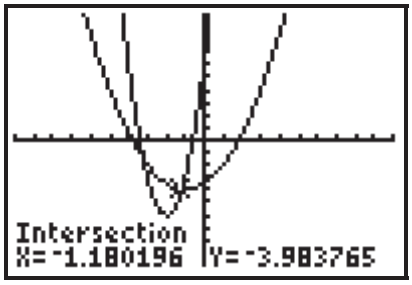
Left Side	Right Side
$5v + 2t + 26$	0
$= 5(-2) + 2(-8) + 26$	
$= 0$	

Left Side = Right Side

d) $n^2 + 2n - 2m - 7 = 0$

$3n^2 + 12n - m + 6 = 0$





From the graph, the solutions are approximately (–3.22, –1.54) and (–1.18, –3.98).

For (–3.22, –1.54):

In $n^2 + 2n - 2m - 7 = 0$

Left Side	Right Side
$n^2 + 2n - 2m - 7$	0
$= (-3.22)^2 + 2(-3.22) - 2(-1.54) - 7$	
$= 0.0084$	
≈ 0	

Left Side = Right Side

In $3n^2 + 12n - m + 6 = 0$

Left Side	Right Side
$3n^2 + 12n - m + 6$	0
$= 3(-3.22)^2 + 12(-3.22) - (-1.54) + 6$	
$= 0.0052$	
≈ 0	

Left Side = Right Side

For (–1.18, –3.98):

In $n^2 + 2n - 2m - 7 = 0$

Left Side	Right Side
$n^2 + 2n - 2m - 7$	0
$= (-1.18)^2 + 2(-1.18) - 2(-3.98) - 7$	
$= -0.0076$	
≈ 0	

Left Side = Right Side

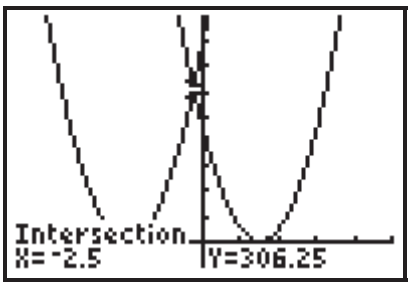
In $3n^2 + 12n - m + 6 = 0$

Left Side
$3n^2 + 12n - m + 6$
$= 3(-1.18)^2 + 12(-1.18) - (-3.98) + 6$
$= -0.0028$
≈ 0

Right Side
0

Left Side = Right Side

e)
$0 = t^2 + 40t - h + 400$
$t^2 = h + 30t - 225$
From the graph, the solution is $(-2.5, 306.25)$.



For $(-2.5, 306.25)$:
In $0 = t^2 + 40t - h + 400$

Left Side
0

Right Side
$t^2 + 40t - h + 400$
$= (-2.5)^2 + 40(-2.5) - 306.25 + 400$
$= 0$

Left Side = Right Side

In $t^2 = h + 30t - 225$

Left Side
t^2
$= (-2.5)^2$
$= 6.25$

Right Side
$h + 30t - 225$
$= 306.25 + 30(-2.5) - 225$
$= 6.25$

Left Side = Right Side

Section 8.1 Page 436 Question 6

The two parabolas have the same vertex, but different a values.
Example: $y = x^2$ and $y = 2x^2$.