Section 9.2 Page 484 Question 3

a) Substitute $x = 4$ into $x^2 - 3x - 10$.

$4^2 - 3(4) - 10$
$= 16 - 12 - 10$
$= -6$

Since $-6 \not> 0$, $x = 4$ is not a solution to $x^2 - 3x - 10 > 0$.

b) Substitute $x = 1$ into $x^2 + 3x - 4$.

$1^2 + 3(1) - 4$
$= 0$

So $x = 1$ is a solution to $x^2 + 3x - 4 \geq 0$.

c) Substitute $x = -2$ into $x^2 + 4x + 3$.

$(-2)^2 + 4(-2) + 3$
$= 4 - 8 + 3$
$= -1$

Since $-1 < 0$, $x = -2$ is a solution to $x^2 + 4x + 3 < 0$.

d) Substitute $x = -3$ into $-x^2 - 5x - 4$.

$-(-3)^2 - 5(-3) - 4$
$= -9 + 15 - 4$
$= 2$

Since $2 > 0$, $x = -2$ is a not a solution to $-x^2 - 5x - 4 \leq 0$.

Section 9.2 Page 485 Question 4

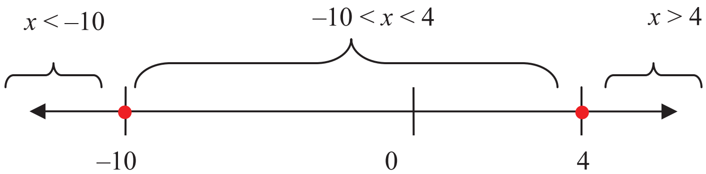
a) Solve $x(x + 6) \geq 40$.

First, determine the roots of the related equation.

$$\begin{aligned} x(x + 6) &= 40 \\ x^2 + 6x - 40 &= 0 \\ (x + 10)(x - 4) &= 0 \end{aligned}$$

$x + 10 = 0$ or $x - 4 = 0$
$x = -10$ or $x = 4$

Plot -10 and 4 on a number line.



Choose test points in each of the three intervals as shown in the table.

Interval	$x < -10$	$-10 < x < 4$	$x > 4$
Test Point	-12	0	5
Substitution	$-12(-12 + 6)$ $= -12(-6)$ $= 72$	$0(0 + 6)$ $= 0$	$5(5 + 6)$ $= 5(11)$ $= 55$
Is $x(x + 6) \geq 40$?	yes	no	yes

The solution set is $\{x \mid x \leq -10 \text{ or } x \geq 4,\ x \in \mathrm{R}\}$.

b) Solve $-x^2 - 14x - 24 < 0$.

First, determine the roots of the related equation.

$-x^2 - 14x - 24 = 0$

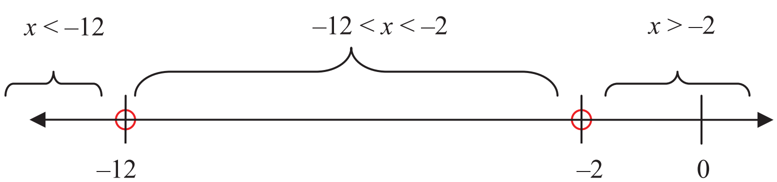
$x^2 + 14x + 24 = 0$

$(x + 12)(x + 2) = 0$

$x + 12 = 0$ or $x + 2 = 0$

$x = -12$ or $x = -2$

Plot –12 and –2 on a number line.



Choose test points in each of the three intervals as shown in the table.

Interval	$x < -12$	$-12 < x < -2$	$x > -2$
Test Point	-20	-5	0
Substitution	$-(20)^2 - 14(20) - 24$ $= -400 - 280 - 24$ $= -704$	$(-5)^2 - 14(-5) - 24$ $= 25 + 70 - 24$ $= 71$	$(0)2 - 14(0) - 24$ $= -24$
Is $-x^2 - 14x - 24 < 0$?	yes	no	yes

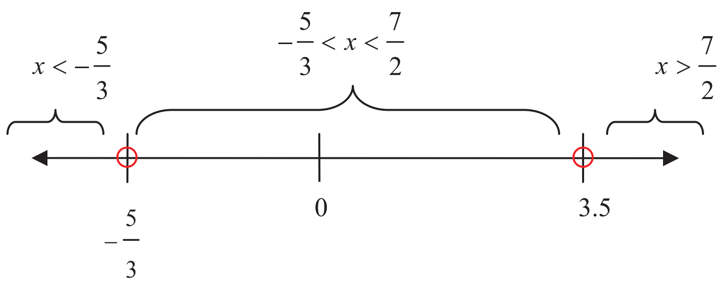
The solution set is $\{x \mid x < -12 \text{ or } x > -2,\ x \in \mathrm{R}\}$.

c) Solve $6x^2 > 11x + 35$.

First, determine the roots of the related equation.

$$6x^2 = 11x + 35$$
$$6x^2 - 11x - 35 = 0$$
$$(3x + 5)(2x - 7) = 0$$
$$3x + 5 = 0 \quad \text{or} \quad 2x - 7 = 0$$
$$x = -\frac{5}{3} \quad \text{or} \quad x = \frac{7}{2}$$

Plot $-\frac{5}{3}$ and $\frac{7}{2}$ on a number line.



Choose test points in each of the three intervals as shown in the table.

Interval	$x < -\frac{5}{3}$	$-\frac{5}{3} < x < \frac{7}{2}$	$x > \frac{7}{2}$
Test Point	–2	0	5
Substitution	$6x^2 = 6(-2)^2 = 24$ $11x + 35 = 11(-2) + 35 = 13$	$6x^2 = 6(0)^2 = 0$ $11x + 35 = 11(0) + 35 = 35$	$6x^2 = 6(5)^2 = 150$ $11x + 35 = 11(5) + 35 = 90$
Is $6x^2 > 11x + 35$?	yes	no	yes

The solution set is $\{x \mid x < -\frac{5}{3} \text{ or } x > \frac{7}{2},\ x \in \mathrm{R}\}$.

d) Solve $8x + 5 \le -2x^2$.

First, determine the roots of the related equation.

$$8x + 5 = -2x^2$$
$$2x^2 + 8x + 5 = 0$$

The equation does not factor. Use the quadratic formula with $a = 2$, $b = 8$, and $c = 5$.

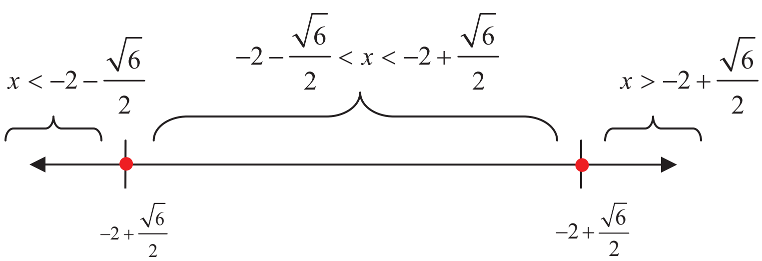
$$x = \frac{-8 \pm \sqrt{8^2 - 4(2)(5)}}{2(2)}$$

$$x = \frac{-8 \pm \sqrt{24}}{4}$$

$$x = -2 \pm \frac{\sqrt{6}}{2}$$

Plot $-2 + \frac{\sqrt{6}}{2}$ and $-2 - \frac{\sqrt{6}}{2}$ on a number line.

The points are approximately –0.775 and –3.22.



Choose test points in each of the three intervals as shown in the table.

Interval	$x < -2 - \frac{\sqrt{6}}{2}$	$-2 - \frac{\sqrt{6}}{2} < x < -2 + \frac{\sqrt{6}}{2}$	$x > -2 + \frac{\sqrt{6}}{2}$
Test Point	–4	–1	0
Substitution	$8x + 5 = 8(-4) + 5$ $= -27$ $-2x^2 = -2(-4)^2$ $= -32$	$8x + 5 = 8(-1) + 5$ $= -3$ $-2x^2 = -2(-1)^2$ $= -2$	$8x + 5 = 8(0) + 5$ $= 5$ $-2x^2 = -2(0)^2$ $= 0$
Is $8x + 5 \le -2x^2$?	no	yes	no

The solution set is $\{x \mid -2 - \frac{\sqrt{6}}{2} \le x \le -2 + \frac{\sqrt{6}}{2},\ x \in \mathrm{R}\}$.

Section 9.2　Page 485　Question 5

a) $x^2 + 3x \le 18$

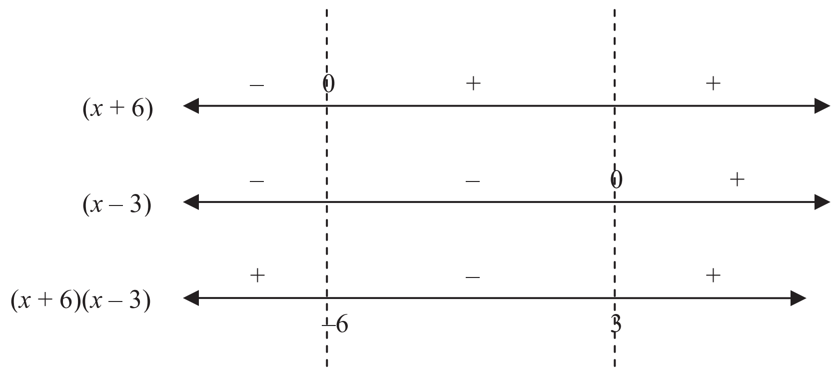
$x^2 + 3x - 18 \le 0$

$(x + 6)(x - 3) \le 0$

Substitute 3 in $(x + 6)$: $3 + 6 = 9$ is positive.

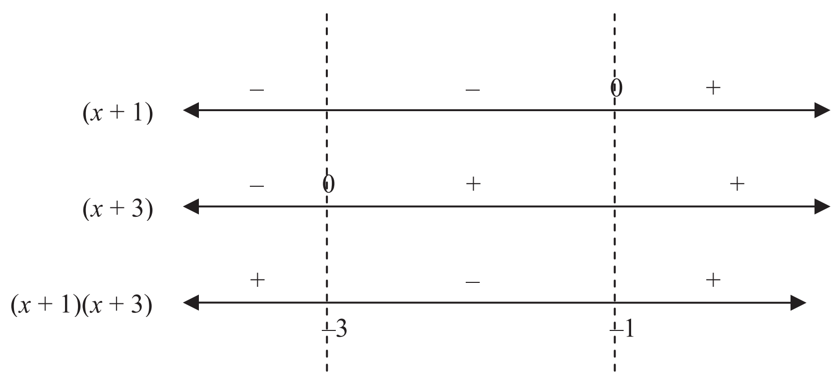
Substitute –6 in $(x - 3)$: $-6 - 3 = -9$ is negative.

The signs of the factors in each interval are shown on the diagram below.



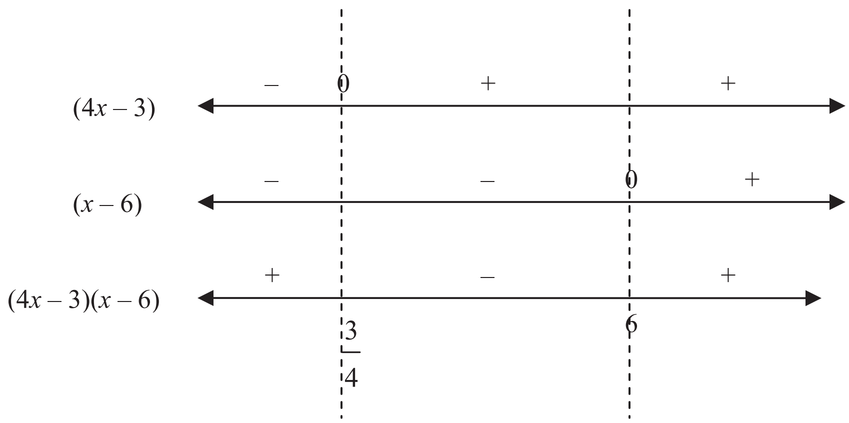
So, the middle interval is where $x^2 + 3x - 18 \le 0$ or $x^2 + 3x \le 18$.
The solution set is $\{x \mid -6 \le x \le 3,\ x \in \mathrm{R}\}$.

b) $x^2 + 3 \ge -4x$
$x^2 + 4x + 3 \ge 0$
$(x + 1)(x + 3) \ge 0$
Substitute -1 in $(x + 3)$: $-1 + 3 = 2$ is positive.
Substitute -3 in $(x + 1)$: $-3 + 1 = -2$ is negative.
The signs of the factors in each interval are shown on the diagram below.



So, the two outer intervals are where $x^2 + 4x + 3 \ge 0$ or $x^2 + 3 \ge -4x$.
The solution set is $\{x \mid x \le -3 \text{ or } x \ge -1,\ x \in \mathrm{R}\}$.

c) $4x^2 - 27x + 18 < 0$
$(4x - 3)(x - 6) < 0$
Substitute $\frac{3}{4}$ in $(x - 6)$: $\frac{3}{4} - 6 = -\frac{21}{4}$ is negative.
Substitute 6 in $(4x - 3)$: $4(6) - 3 = 21$ is positive.
The signs of the factors in each interval are shown on the diagram below.



So, the middle interval is where $4x^2 - 27x + 18 < 0$.

The solution set is $\{x \mid \frac{3}{4} < x < 6,\ x \in \mathrm{R}\}$.

d) $-6x \geq x^2 - 16$

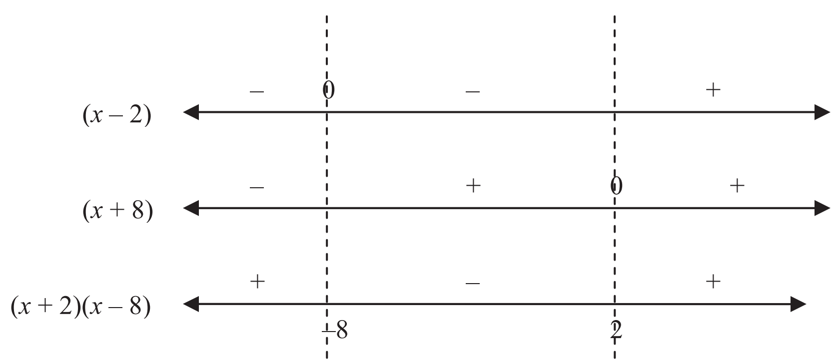
$x^2 + 6x - 16 \leq 0$

$(x - 2)(x + 8) \leq 0$

Substitute –8 in $(x - 2)$: $-8 - 2 = -10$ is negative.

Substitute 2 in $(x + 8)$: $2 + 8 = 10$ is positive.

The signs of the factors in each interval are shown on the diagram below.



So, the middle interval is where $x^2 + 6x - 16 \leq 0$ or $-6x \geq x^2 - 16$.

The solution set is $\{x \mid -8 \leq x \leq 2,\ x \in \mathrm{R}\}$.

Section 9.2 Page 485 Question 6

a) $x^2 - 2x - 15 < 0$

$(x - 5)(x + 3) < 0$

Case 1: The first factor is negative and the second factor is positive.

$x - 5 < 0$ and $x + 3 > 0$

$x < 5$ and $x > -3$

These two inequalities are true for all points between –3 and 5.

Case 2: The first factor is positive and the second factor is negative.

$x - 5 > 0$ and $x + 3 < 0$

$x > 5$ and $x < -3$

The pair of conditions is never true.

The solution set is $\{x \mid -3 < x < 5,\ x \in \mathrm{R}\}$.

b) $x^2 + 13x > -12$

$x^2 + 13x + 12 > 0$

$(x + 12)(x + 1) > 0$

Case 1: Both factors are positive.

$x + 12 > 0$ and $x + 1 > 0$

$x > -12$ and $x > -1$

Both conditions are true when $x > -1$.

Case 2: Both factors are negative.

$x + 12 < 0$ and $x + 1 < 0$

$x < -12$ and $x < -1$

Both conditions are true when $x < -12$.

The solution set is $\{x \mid x < -12 \text{ or } x > -1,\ x \in \mathrm{R}\}$.

c) $-x^2 + 2x + 5 \le 0$

$x^2 - 2x - 5 \ge 0$

Use the quadratic formula to solve $x^2 - 2x - 5 = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-5)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{24}}{2}$$

$$x = 1 \pm \sqrt{6}$$

So, the inequality has two factors $(x - 1 - \sqrt{6})(x - 1 + \sqrt{6})$.

For their product to be greater than or equal to zero, both factors must be the same sign.

Case 1: Both factors are positive.

$x - 1 - \sqrt{6} \ge 0$ and $x - 1 + \sqrt{6} \ge 0$

$x \ge 1 + \sqrt{6}$ and $x \ge 1 - \sqrt{6}$

These two inequalities are both true when $x \ge 1 + \sqrt{6}$.

Case 2: Both factors are negative.

$x - 1 - \sqrt{6} \le 0$ and $x - 1 + \sqrt{6} \le 0$

$x \le 1 + \sqrt{6}$ and $x \le 1 - \sqrt{6}$

The pair of conditions is true when $x \le 1 - \sqrt{6}$.

The solution set is $\{x \mid x \le 1 - \sqrt{6}$ or $x \ge 1 + \sqrt{6}$, $x \in \mathrm{R}\}$.

d) $2x^2 \ge 8 - 15x$

$2x^2 + 15x - 8 \ge 0$

$(2x - 1)(x + 8) \ge 0$

Case 1: Both factors are positive.

$2x - 1 \ge 0$ and $x + 8 \ge 0$

$x \ge \frac{1}{2}$ and $x \ge -8$

Both conditions are true when $x \ge \frac{1}{2}$.

Case 2: Both factors are negative.

$2x - 1 \le 0$ and $x + 8 \le 0$

$x \le \frac{1}{2}$ and $x \le -8$

Both conditions are true when $x \le -8$.

The solution set is $\{x \mid x \le -8$ or $x \ge \frac{1}{2}$, $x \in \mathrm{R}\}$.

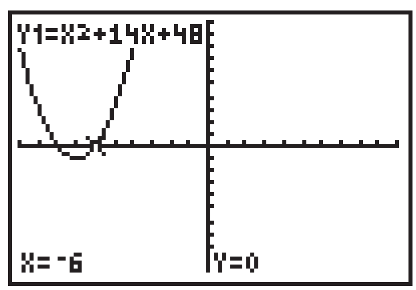
Section 9.2 Page 485 Question 7

a) $x^2 + 14x + 48 \le 0$

Graph $y = x^2 + 14x + 48$.

The graph is below the x-axis between -8 and -6.

The solution set is $\{x \mid -8 \le x \le -6$, $x \in \mathrm{R}\}$.



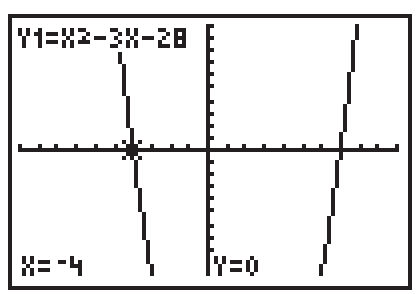
b) $x^2 \ge 3x + 28$

$x^2 - 3x - 28 \ge 0$

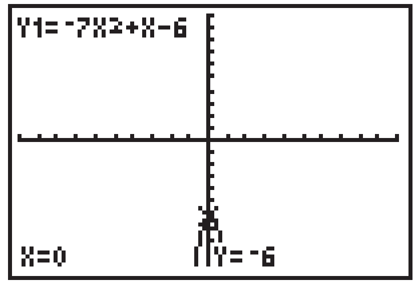
Graph $y = x^2 - 3x - 28$.

The graph is above the x-axis when values of x are less than -4 and greater than 7.

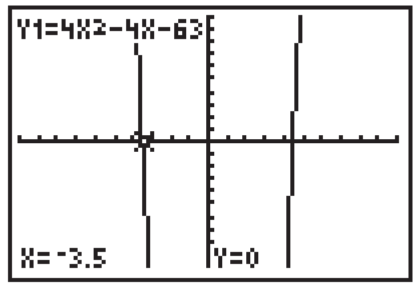
The solution set is $\{x \mid x \le -4$ or $x \ge 7$, $x \in \mathrm{R}\}$.



c) $-7x^2 + x - 6 \geq 0$
Graph $y = -7x^2 + x - 6$.
The graph is never above the x-axis.
There is no solution for $-7x^2 + x - 6 \geq 0$.



d) $4x(x - 1) > 63$
$4x^2 - 4x - 63 > 0$
Graph $y = 4x^2 - 4x - 63$.
Factor to determine the zeros.
$0 = (2x - 9)(2x + 7)$
The zeros are 4.5 and –3.5.
The graph is above the x-axis to the left of –3.5 and to the right of 4.5.
The solution set is $\{x \mid x < -3.5 \text{ or } x > 4.5,\ x \in \mathrm{R}\}$.



Section 9.2 Page 485 Question 8

Methods may vary.
a) $x^2 - 10x + 16 < 0$
Use case analysis because the expression is easy to factor.
$(x - 8)(x - 2) < 0$
Case 1: The first factor is negative and the second factor is positive.
$x - 8 < 0$ and $x - 2 > 0$
$x < 8$ and $x > 2$
These inequalities are true for values of x between 2 and 8.
Case 2: The first factor is positive and the second factor is negative.
$x - 8 > 0$ and $x - 2 < 0$
$x > 8$ and $x < 2$
Both inequalities are never true.
The solution set is $\{x \mid 2 < x < 8,\ x \in \mathrm{R}\}$.

b) $12x^2 - 11x - 15 \geq 0$
Use case analysis because the expression can be factored.
$(3x - 5)(4x + 3) \geq 0$
Case 1: Both factors are positive.
$3x - 5 \geq 0$ and $4x + 3 \geq 0$
$x \geq \frac{5}{3}$ and $x \geq -\frac{3}{4}$