Section 8.2 Solving Systems of Equations Algebraically

Section 8.2 Page 451 Question 1

In $k + p = 12$:
Left Side
$k + p$
$= 5 + 7$
$= 12$
$=$ Right Side

In $4k^2 - 2p = 86$:
Left Side
$4k^2 - 2p$
$= 4(5)^2 - 2(7)$
$= 86$
$=$ Right Side

So, (5, 7) is a solution.

Section 8.2 Page 451 Question 2

In $18w^2 - 16z^2 = -7$:

$$\begin{aligned}\text{Left Side} &= 18\left(\frac{1}{3}\right)^2 - 16\left(\frac{3}{4}\right)^2\\ &= 18\left(\frac{1}{9}\right) - 16\left(\frac{9}{16}\right)\\ &= 2 - 9\\ &= -7\\ &= \text{Right Side}\end{aligned}$$

In $144w^2 + 48z^2 = 43$:

$$\begin{aligned}\text{Left Side} &= 144\left(\frac{1}{3}\right)^2 + 48\left(\frac{3}{4}\right)^2\\ &= 16 + 27\\ &= 43\\ &= \text{Right Side}\end{aligned}$$

So, $\left(\frac{1}{3}, \frac{3}{4}\right)$ is a solution.

Section 8.2 Page 451 Question 3

a) $x^2 - y + 2 = 0$ ①
$4x = 14 - y$ ②

From ②, $y = 14 - 4x$. Substitute into ①.
$x^2 - (14 - 4x) + 2 = 0$
$x^2 - 14 + 4x + 2 = 0$
$x^2 + 4x - 12 = 0$
$(x + 6)(x - 2) = 0$
$x = -6$ or $x = 2$
Substitute into ②.

When $x = -6$:
$4x = 14 - y$
$4(-6) = 14 - y$
$y = 14 + 24$
$y = 38$

When $x = 2$:
$4x = 14 - y$
$4(2) = 14 - y$
$y = 14 - 8$
$y = 6$

Verify (–6, 38).

In $x^2 - y + 2 = 0$:
Left Side $= (-6)^2 - 38 + 2$
$= 36 - 38 + 2$
$= 0$
$=$ Right Side

In $4x = 14 - y$:
Left Side $= 4(-6)$
$= -24$
Right Side $= 14 - 38$
$= -24$
Left Side = Right Side

Verify (2, 6).
In $x^2 - y + 2 = 0$:
Left Side $= 2^2 - 6 + 2$
$= 0$
$=$ Right Side

In $4x = 14 - y$:
Left Side $= 4(2)$
$= 8$
Right Side $= 14 - 6$
$= 8$
Left Side = Right Side

Both solutions, (–6, 38) and (2, 6), check.

b) $2x^2 - 4x + y = 3$ ①
$4x - 2y = -7$ ②
From ②, $y = 2x + 3.5$. Substitute into ①.
$2x^2 - 4x + 2x + 3.5 = 3$
$2x^2 - 2x + 0.5 = 0$
$4x^2 - 4x + 1 = 0$
$(2x - 1)(2x - 1) = 0$
$x = 0.5$
Substitute into ②.
$4x - 2y = -7$
$4(0.5) - 2y = -7$
$2 + 7 = 2y$
$y = 4.5$

Verify (0.5, 4.5).
In $2x^2 - 4x + y = 3$:
Left Side $= 2(0.5)^2 - 4(0.5) + 4.5$
$= 3$
$=$ Right Side

In $4x - 2y = -7$:
Left Side $= 4(0.5) - 2(4.5)$
$= -7$
$=$ Right Side

The solution (0.5, 4.5) checks.

c) $7d^2 + 5d - t - 8 = 0$ ①
$10d - 2t = -40$ ②
From ②, $t = 5d + 20$. Substitute into ①.
$7d^2 + 5d - (5d + 20) - 8 = 0$
$7d^2 - 28 = 0$
$7(d - 2)(d + 2) = 0$
$d = 2$ or $d = -2$
Substitute into ②.

When $d = 2$:
$10d - 2t = -40$
$10(2) - 2t = -40$

When $d = -2$:
$10d - 2t = -40$
$10(-2) - 2t = -40$

$60 = 2t$
$t = 30$

$-2t = -20$
$t = 10$

Verify (2, 30).

In $7d^2 + 5d - t - 8 = 0$:

$$\begin{aligned}\text{Left Side} &= 7(2)^2 + 5(2) - 30 - 8\\ &= 28 + 10 - 38\\ &= 0\\ &= \text{Right Side}\end{aligned}$$

In $10d - 2t = -40$:

$$\begin{aligned}\text{Left Side} &= 10(2) - 2(30)\\ &= 20 - 60\\ &= -40\\ &= \text{Right Side}\end{aligned}$$

Verify (–2, 10).

In $7d^2 + 5d - t - 8 = 0$:

$$\begin{aligned}\text{Left Side} &= 7(-2)^2 + 5(-2) - 10 - 8\\ &= 28 - 10 - 18\\ &= 0\\ &= \text{Right Side}\end{aligned}$$

In $10d - 2t = -40$:

$$\begin{aligned}\text{Left Side} &= 10(-2) - 2(10)\\ &= -20 - 20\\ &= -40\\ &= \text{Right Side}\end{aligned}$$

Both solutions, (2, 30) and (–2, 10), check.

d) $3x^2 + 4x - y - 8 = 0$ ①
$y + 3 = 2x^2 + 4x$ ②

From ②, $y = 2x^2 + 4x - 3$. Substitute in ①.

$3x^2 + 4x - (2x^2 + 4x - 3) - 8 = 0$
$x^2 - 5 = 0$
$x = \pm\sqrt{5}$
$x \approx 2.24$ or $x \approx -2.24$

Substitute in ②.

When $x = \sqrt{5}$:
$y + 3 = 2x^2 + 4x$
$y + 3 = 2(\sqrt{5})^2 + 4(\sqrt{5})$
$y = 7 + 4\sqrt{5}$
$y \approx 15.94$

When $x = -\sqrt{5}$:
$y + 3 = 2x^2 + 4x$
$y + 3 = 2(-\sqrt{5})^2 + 4(-\sqrt{5})$
$y = 7 - 4\sqrt{5}$
$y \approx -1.94$

Verify $(\sqrt{5}, 7 + 4\sqrt{5})$.

In $3x^2 + 4x - y - 8 = 0$:

$$\begin{aligned}\text{Left Side} &= 3(\sqrt{5})^2 + 4(\sqrt{5}) - (7 + 4\sqrt{5}) - 8\\ &= 15 - 15\\ &= 0\\ &= \text{Right Side}\end{aligned}$$

In $y + 3 = 2x^2 + 4x$:

Left Side	Right Side
$= 7 + 4\sqrt{5} + 3$	$= 2(\sqrt{5})^2 + 4(\sqrt{5})$
$= 10 + 4\sqrt{5}$	$= 10 + 4\sqrt{5}$

Left Side = Right Side

Verify $(-\sqrt{5}, 7 - 4\sqrt{5})$.

In $3x^2 + 4x - y - 8 = 0$:

$$\begin{aligned}\text{Left Side} &= 3(-\sqrt{5})^2 + 4(-\sqrt{5}) - (7 - 4\sqrt{5}) - 8\\ &= 15 - 15\\ &= 0\end{aligned}$$

In $y + 3 = 2x^2 + 4x$:

Left Side	Right Side
$= 7 - 4\sqrt{5} + 3$	$= 2(-\sqrt{5})^2 + 4(-\sqrt{5})$
$= 10 - 4\sqrt{5}$	$= 10 - 4\sqrt{5}$

$= \text{Right Side}$ Left Side = Right Side

The two solutions check. The approximate answers are (2.24, 15.94) and (–2.24, –1.94).

e) $y + 2x = x^2 - 6$ ①
$x + y - 3 = 2x^2$ ②
From ②, $y = 2x^2 - x + 3$. Substitute in ①.
$2x^2 - x + 3 + 2x = x^2 - 6$
$x^2 + x + 9 = 0$
Use the quadratic formula with $a = 1$, $b = 1$, and $c = 9$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(3)}}{2(2)}$$

$$x = \frac{1 \pm \sqrt{-23}}{4}$$

This system has no real solutions.

Section 8.2 Page 452 Question 4

a) $6x^2 - 3x = 2y - 5$ ①
$2x^2 + x = y - 4$ ②
Multiply ② by –2.
$6x^2 - 3x = 2y - 5$ ①
$-4x^2 - 2x = -2y + 8$ ③
Add ① + ③.
$2x^2 - 5x = 3$
$2x^2 - 5x - 3 = 0$
$(2x + 1)(x - 3) = 0$
$x = -0.5$ or $x = 3$
Substitute in ②.

When $x = -0.5$:
$2x^2 + x = y - 4$
$2(0.5)^2 + (-0.5) + 4 = y$
$y = 4$

When $x = 3$:
$2x^2 + x = y - 4$
$2(3)^2 + 3 + 4 = y$
$y = 25$

Verify (–0.5, 4).
In ①:
Left Side $= 6x^2 - 3x$
$= 6(-0.5)^2 - 3(-0.5)$
$= 3$

Right Side $= 2y - 5$
$= 2(4) - 5$
$= 3$

Left Side = Right Side

In ②:

Left Side = $2x^2 + x$ | Right Side = $y - 4$
$= 2(–0.5)^2 + (–0.5)$ | $= 4 – 4$
$= 0$ | $= 0$

Left Side = Right Side

Verify (3, 25).

In ①:

Left Side = $6x^2 - 3x$ | Right Side = $2y – 5$
$= 6(3)^2 – 3(3)$ | $= 2(25) – 5$
$= 45$ | $= 45$

Left Side = Right Side

In ②:

Left Side = $2x^2 + x$ | Right Side = $y - 4$
$= 2(3)^2 + 3$ | $= 25 – 4$
$= 21$ | $= 21$

Left Side = Right Side

Both solutions, (–0.5, 4) and (3, 25), check.

b) $x^2 + y = 8x + 19$ ①
$x^2 - y = 7x - 11$ ②

Add ① + ②.

$2x^2 = 15x + 8$
$2x^2 – 15x – 8 = 0$
$(2x + 1)(x – 8) = 0$
$x = –0.5$ or $x = 8$

Substitute in ①.

When $x = –0.5$:
$y = 8x + 19 – x^2$
$y = 8(–0.5) + 19 – (–0.5)^2$
$y = 14.75$

When $x = 8$:
$y = 8x + 19 – x^2$
$y = 8(8) + 19 – 8^2$
$y = 19$

Verify (–0.5, 14.75).

In ①:

Left Side = $x^2 + y$ | Right Side = $8x + 19$
$= (–0.5)^2 + 14.75$ | $= 8(–0.5) + 19$
$= 15$ | $= 15$

Left Side = Right Side

In ②:

Left Side = $x^2 – y$ | Right Side = $7x – 11$
$= (–0.5)^2 + 14.75$ | $= 7(–0.5) – 11$
$= –14.5$ | $= –14.5$

Left Side = Right Side

Verify (8, 19).

In ①:

Left Side $= x^2 + y$
$= 8^2 + 19$
$= 83$

Right Side $= 8x + 19$
$= 8(8) + 19$
$= 83$

Left Side = Right Side

In ②:

Left Side $= x^2 - y$
$= 8^2 - 19$
$= 45$

Right Side $= 7x - 11$
$= 7(8) - 11$
$= 45$

Left Side = Right Side

Both solutions, (–0.5, 14.75) and (8, 19), check.

c) $2p^2 = 4p - 2m + 6$ ①

$5m + 8 = 10p + 5p^2$ ②

Multiply ① by 5 and ② by –2, and rearrange terms.

$10p^2 - 20p - 30 = -10m$ ③

$10p^2 + 20p - 16 = 10m$ ④

Add ③ + ④.

$20p^2 - 46 = 0$

$p = \pm\sqrt{2.3}$

$p \approx 1.52$ or $p \approx -1.52$

Substitute in ②.

When $p = \sqrt{2.3}$:

$5m = 10p + 5p^2 - 8$

$5m = 10(\sqrt{2.3}) + 5(\sqrt{2.3})^2 - 8$

$5m = 10\sqrt{2.3} + 11.5 - 8$

$m = 2\sqrt{2.3} + 0.7$

$m \approx 3.73$

When $p = -\sqrt{2.3}$:

$5m = 10p + 5p^2 - 8$

$5m = 10(-\sqrt{2.3}) + 5(-\sqrt{2.3})^2 - 8$

$5m = -10\sqrt{2.3} + 3.5$

$m = -2\sqrt{2.3} + 0.7$

$m \approx -2.33$

Verify ($\sqrt{2.3}$, $2\sqrt{2.3} + 0.7$).

In ①:

Left Side $= 2p^2$
$= 2(\sqrt{2.3})^2$
$= 4.6$

Right Side $= 4p - 2m + 6$
$= 4(\sqrt{2.3}) - 2(2\sqrt{2.3} + 0.7) + 6$
$= 4.6$

Left Side = Right Side

In ②:

Left Side $= 5m + 8$
$= 5(2\sqrt{2.3} + 0.7) + 8$
$= 10\sqrt{2.3} + 3.5 + 8$
$= 10\sqrt{2.3} + 11.5$

Right Side $= 10p + 5p^2$
$= 10(\sqrt{2.3}) + 5(\sqrt{2.3})^2$
$= 10\sqrt{2.3} + 11.5$

Left Side = Right Side

Verify $(-\sqrt{2.3}, -2\sqrt{2.3} + 0.7)$.
In ①:

Left Side $= 2p^2$
$= 2(-\sqrt{2.3})^2$
$= 4.6$

Right Side $= 4p - 2m + 6$
$= 4(-\sqrt{2.3}) - 2(-2\sqrt{2.3} + 0.7) + 6$
$= 4.6$

Left Side = Right Side

In ②:

Left Side $= 5m + 8$
$= 5(-2\sqrt{2.3} + 0.7) + 8$
$= -10\sqrt{2.3} + 3.5 + 8$
$= -10\sqrt{2.3} + 11.5$

Right Side $= 10p + 5p^2$
$= 10(-\sqrt{2.3}) + 5(-\sqrt{2.3})^2$
$= -10\sqrt{2.3} + 11.5$

Left Side = Right Side

Both solutions, $(\sqrt{2.3}, 2\sqrt{2.3} + 0.7)$ and $(-\sqrt{2.3}, -2\sqrt{2.3} + 0.7)$, check. The solutions are approximately (1.52, 3.73) and (–1.52, –2.33).

d) $9w^2 + 8k = -14$ ①
$w^2 + k = -2$ ②
Multiply ② by –8.
$-8w^2 - 8k = 16$ ③
Add ① and ③.
$w^2 = 2$
$w = \pm\sqrt{2}$
Substitute in ②.

When $w = \sqrt{2}$:
$w^2 + k = -2$
$(\sqrt{2})^2 + k = -2$
$k = -4$

When $w = -\sqrt{2}$:
$w^2 + k = -2$
$(-\sqrt{2})^2 + k = -2$
$k = -4$

Verify $(\sqrt{2}, -4)$.

In ①:
Left Side $= 9w^2 + 8k$
$= 9(\sqrt{2})^2 + 8(-4)$
$= -14$
= Right Side

In ②:
Left Side $= w^2 + k$
$= (\sqrt{2})^2 + (-4)$
$= -2$
= Right Side

Verify $(-\sqrt{2}, -4)$.

In ①:
Left Side $= 9w^2 + 8k$
$= 9(-\sqrt{2})^2 + 8(-4)$
$= -14$
= Right Side

In ②:
Left Side $= w^2 + k$
$= (-\sqrt{2})^2 + (-4)$
$= -2$
= Right Side

Both solutions, $(\sqrt{2}, -4)$ and $(-\sqrt{2}, -4)$, check. The solutions are approximately (1.41, –4) and (–1.41, –4).

e) $4h^2 - 8t = 6$ ①
$6h^2 - 9 = 12t$ ②
Multiply ① by 3 and ② by 2. Align like terms.
$12h^2 - 24t = 18$ ③
$12h^2 - 24t = 18$ ④
Both original equations are equivalent. They have an infinite number of solutions.

Section 8.2 Page 452 Question 5

a) $y - 1 = -\frac{7}{8}x$ ①
$3x^2 + y = 8x - 1$ ②
I will use the substitution method because I can easily obtain an expression for y from equation ①.
From ① $y = 1 - \frac{7}{8}x$. Substitute in ②.

$$3x^2 + 1 - \frac{7}{8}x = 8x - 1$$
$$24x^2 + 16 - 7x - 64x = 0$$
$$24x^2 - 71x + 16 = 0$$

Use the quadratic formula with $a = 24$, $b = -71$, and $c = 16$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-71) \pm \sqrt{(-71)^2 - 4(24)(16)}}{2(24)}$$
$$x = \frac{71 \pm \sqrt{3505}}{48}$$

$x \approx 2.71$ or $x \approx 0.25$

Substitute in ①.

When $x = \frac{71 + \sqrt{3505}}{48}$:
$$y = 1 - \frac{7}{8}\left(\frac{71 + \sqrt{3505}}{48}\right)$$
$$y = \frac{-113 - 7\sqrt{3505}}{384}$$
$y \approx -1.37$

When $x = \frac{71 - \sqrt{3505}}{48}$:
$$y = 1 - \frac{7}{8}\left(\frac{71 - \sqrt{3505}}{48}\right)$$
$$y = \frac{-113 + 7\sqrt{3505}}{384}$$
$y \approx 0.78$

The solutions are approximately (2.71, –1.37) and (0.25, 0.78).

b) $8x^2 + 5y = 100$ ①
$6x^2 - x - 3y = 5$ ②
I will use the elimination method because I can eliminate y by adding, if I first multiply ① by 3 and ② by 5.