Chapter 4

Completing the Square ($a = 1$)

To solve the following quadratic equation by completing the square, what constant term should you add?

The Constant Term

a) $x^2 - 8x + 13 = 0$

$x^2 - 8x + c = -13 + c$

b) $x^2 + 6x + 1 = 0$

$x^2 + 6x + c = -1 + c$

c) $x^2 + 4x + 3 = 0$

$x^2 + 4x + c = -3 + c$

Solve a Quadratic Equation by Completing the Square When $a = 1$

Solve $x^2 - 21 = -10x$ by completing the square. Express your answers to the nearest tenth.

Can you solve this equation by factoring? Explain.

Solution

$$x^2 - 21 = -10x$$
$$x^2 + 10x = 21$$
$$x^2 + 10x + 25 = 21 + 25$$
$$(x + 5)^2 = 46$$
$$x + 5 = \pm\sqrt{46}$$

Solve for x.

$x + 5 = \sqrt{46}$ or $x + 5 = -\sqrt{46}$

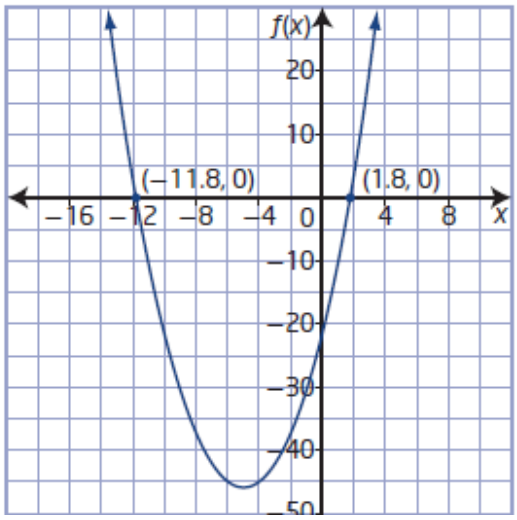
$x = -5 + \sqrt{46}$ $x = -5 - \sqrt{46}$

$x = 1.7823\ldots$ $x = -11.7823\ldots$

The exact roots are $-5 + \sqrt{46}$ and $-5 - \sqrt{46}$.
The roots are 1.8 and -11.8, to the nearest tenth.

You can also see the solutions to this equation graphically as the x-intercepts of the graph of the function $f(x) = x^2 + 10x - 21$.

These occur at approximately $(-11.8, 0)$ and $(1.8, 0)$ and have values of -11.8 and 1.8, respectively.



Solve a Quadratic Equation by Completing the Square When $a \neq 1$

Determine the roots of $-2x^2 - 3x + 7 = 0$, to the nearest hundredth. Then, use technology to verify your answers.

Solution

$$-2x^2 - 3x + 7 = 0$$

$$x^2 + \frac{3}{2}x - \frac{7}{2} = 0$$ Divide both sides by a factor of -2.

$$x^2 + \frac{3}{2}x = \frac{7}{2}$$ Isolate the variable terms on the left side.

$$x^2 + \frac{3}{2}x + \frac{9}{16} = \frac{7}{2} + \frac{9}{16}$$ Why is $\frac{9}{16}$ added to both sides?

$$\left(x + \frac{3}{4}\right)^2 = \frac{65}{16}$$

$$x + \frac{3}{4} = \pm\sqrt{\frac{65}{16}}$$ Solve for x.

$$x = -\frac{3}{4} \pm \frac{\sqrt{65}}{4}$$

$$x = \frac{-3 \pm \sqrt{65}}{4}$$

The exact roots are $\frac{-3 + \sqrt{65}}{4}$ and $\frac{-3 - \sqrt{65}}{4}$.

The roots are 1.27 and -2.77, to the nearest hundredth.

Chapter 4

Completing the Square (a ≠ 1)

To solve the following quadratic equation by completing the square, what common factor should you divide by and what constant term should you add?

	The Common Factor	The Constant Term
a) $0.5x^2 - 4x + 3 = 0$		
b) $3x^2 + 6x + 1 = 0$		
c) $-2x^2 + 4x + 3 = 0$		

Apply Completing the Square

A defender kicks a soccer ball away from her own goal. The path of the kicked soccer ball can be approximated by the quadratic function $h(x) = -0.06x^2 + 3.168x - 35.34$, where x is the horizontal distance travelled, in metres, from the goal line and h is the height, in metres.

a) You can determine the distance the soccer ball is from the goal line by solving the corresponding equation, $-0.06x^2 + 3.168x - 35.34 = 0$. How far is the soccer ball from the goal line when it is kicked? Express your answer to the nearest tenth of a metre.

b) How far does the soccer ball travel before it hits the ground?

Solution

a) Solve the equation $-0.06x^2 + 3.168x - 35.34 = 0$ by completing the square.

$$-0.06x^2 + 3.168x - 35.34 = 0$$

$$x^2 - 52.8x + 589 = 0$$ Divide both sides by a common factor of -0.06.

$$x^2 - 52.8x = -589$$ Isolate the variable terms on the left side.

$$x^2 - 52.8x + \left(\frac{52.8}{2}\right)^2 = -589 + \left(\frac{52.8}{2}\right)^2$$ Complete the square on the left side.

$$x^2 - 52.8x + 696.96 = -589 + 696.96$$

$$(x - 26.4)^2 = 107.96$$

$$x - 26.4 = \pm\sqrt{107.96}$$ Take the square root of both sides.

Solve for x.

$x - 26.4 = \sqrt{107.96}$ or $x - 26.4 = -\sqrt{107.96}$

$x = 26.4 + \sqrt{107.96}$ $x = 26.4 - \sqrt{107.96}$

$x = 36.7903\ldots$ $x = 16.0096\ldots$

Chapter 4

Solving Quadratic Equations

As a class, discuss the conditions under which each method would be preferred over the others in solving a quadratic equation.

a) graphing the corresponding quadratic function
b) determining the square roots
c) factoring
d) completing the square
e) using the quadratic formula