Example 1

Solve a System of Linear-Quadratic Equations Algebraically

a) Solve the following system of equations.

$5x - y = 10$

$x^2 + x - 2y = 0$

b) Verify your solution.

Solution

a) Method 1: Use Substitution

Since the quadratic term is in the variable x, solve the linear equation for y.

Solve the linear equation for y.

$5x - y = 10$

$y = 5x - 10$

Why is it easier to solve the first equation for y?

Substitute $5x - 10$ for y in the quadratic equation and simplify.

$x^2 + x - 2y = 0$

$x^2 + x - 2(5x - 10) = 0$

$x^2 - 9x + 20 = 0$

Solve the quadratic equation by factoring.

$(x - 4)(x - 5) = 0$

$x = 4$ or $x = 5$

Substitute these values into the original linear equation to determine the corresponding values of y.

When $x = 4$:

$5x - y = 10$

$5(4) - y = 10$

$y = 10$

When $x = 5$:

$5x - y = 10$

$5(5) - y = 10$

$y = 15$

Why substitute into the linear equation rather than the quadratic?

The two solutions are (4, 10) and (5, 15).

Method 2: Use Elimination

Align the terms with the same degree.

Since the quadratic term is in the variable x, eliminate the y-term.

$5x - y = 10$ ①

$x^2 + x - 2y = 0$ ②

Multiply ① by -2 so that there is an opposite term to $-2y$ in ①.

$-2(5x - y) = -2(10)$

$-10x + 2y = -20$ ③

Add ③ and ① to eliminate the y-terms.

$$\begin{array}{r} -10x + 2y = -20 \\ x^2 + x - 2y = 0 \\ \hline x^2 - 9x = -20 \end{array}$$

Then, solve the equation $x^2 - 9x + 20 = 0$ by factoring, as in the substitution method above, to obtain the two solutions (4, 10) and (5, 15).

What do the two solutions tell you about the appearance of the graphs of the two equations?

b) To verify the solutions, substitute each ordered pair into the original equations.

How could you verify the solutions using technology?

Verify the solution (4, 10):

Left Side	Right Side
$5x - y$	10
$= 5(4) - 10$	
$= 20 - 10$	
$= 10$	
Left Side = Right Side	

Left Side	Right Side
$x^2 + x - 2y$	0
$= 4^2 + 4 - 2(10)$	
$= 16 + 4 - 20$	
$= 0$	
Left Side = Right Side	

Verify the solution (5, 15):

Left Side	Right Side
$5x - y$	10
$= 5(5) - 15$	
$= 25 - 15$	
$= 10$	
Left Side = Right Side	

Left Side	Right Side
$x^2 + x - 2y$	0
$= 5^2 + 5 - 2(15)$	
$= 25 + 5 - 30$	
$= 0$	
Left Side = Right Side	

Both solutions are correct.

The two solutions are (4, 10) and (5, 15).