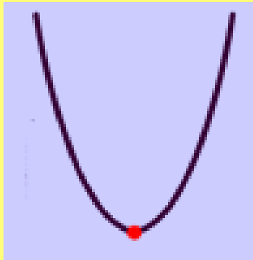
SUMMARY



Quadratic Functions

- in standard form is written as

$$y = ax^2 + bx + c \qquad (a \neq 0)$$

- graph is a parabola
- if a is positive, the parabola opens up (minimum)
- if a is negative, the parabola opens down (maximum)
- changing b and c changes the axis of symmetry
- the value of c is the parabola's y-intercept

http://www.mathopenref.com/quadraticexplorer.html

LEARN ABOUT the Math

Nicolina plays on her school's volleyball team. At a recent match, her Nonno, Marko, took some time-lapse photographs while she warmed up. He set his camera to take pictures every 0.25 s. He started his camera at the moment the ball left her arms during a bump and stopped the camera at the moment that the ball hit the floor. Marko wanted to capture a photo of the ball at its greatest height. However, after looking at the photographs, he could not be sure that he had done so. He decided to place the information from his photographs in a table of values.

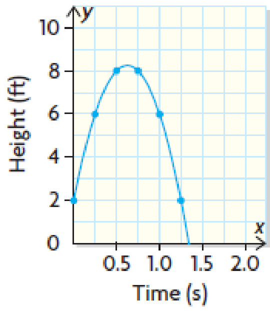
From his photographs, Marko observed that Nicolina struck the ball at a height of 2 ft above the ground. He also observed that it took about 1.25 s for the ball to reach the same height on the way down.

Time (s)	Height (ft)
0.00	2
0.25	6
0.50	8
0.75	8
1.00	6
1.25	2

? When did the volleyball reach its greatest height?

EXAMPLE 1 Using symmetry to estimate the coordinates of the vertex

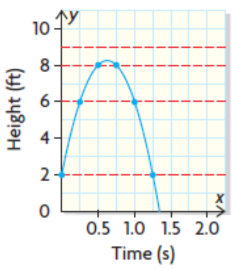
Marko's Solution



I plotted the points from my table, and then I sketched a graph that passed through all the points.

The graph looked like a parabola, so I concluded that the relation is probably quadratic.

vertex
The point at which the quadratic function reaches its maximum or minimum value.



I knew that I could draw horizontal lines that would intersect the parabola at two points, except at the **vertex**, where a horizontal line would intersect the parabola at only one point.

Using a ruler, I drew horizontal lines and estimated that the coordinates of the vertex are around (0.6, 8.2).

This means that the ball reached maximum height at just over 8 ft, about 0.6 s after it was launched.

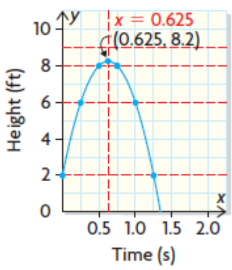
Equation of the axis of symmetry:

$$x = \frac{0 + 1.25}{2}$$

$$x = 0.625$$

I used points that have the same y-value, (0, 2) and (1.25, 2), to determine the equation of the **axis of symmetry**. I knew that the axis of symmetry must be the same distance from each of these points.

axis of symmetry
A line that separates a 2-D figure into two identical parts. For example, a parabola has a vertical axis of symmetry passing through its vertex.



EXAMPLE 2 Reasoning about the maximum value of a quadratic function

Some children are playing at the local splash pad. The water jets spray water from ground level. The path of water from one of these jets forms an arch that can be defined by the function

$$f(x) = -0.12x^2 + 3x$$

where x represents the horizontal distance from the opening in the ground in feet and $f(x)$ is the height of the sprayed water, also measured in feet. What is the maximum height of the arch of water, and how far from the opening in the ground can the water reach?

Manuel's Solution

$f(x) = -0.12x^2 + 3x$

I knew that the degree of the function is 2, so the function is quadratic. The arch must be a parabola.

I also knew that the coefficient of x^2, a, is negative, so the parabola opens down. This means that the function has a **maximum value**, associated with the y-coordinate of the vertex.

$f(0) = 0$

I started to create a table of values by determining the y-intercept. I knew that the constant, zero, is the y-coordinate of the y-intercept. This confirms that the stream of water shoots from ground level.

$f(1) = -0.12(1)^2 + 3(1)$
$f(1) = -0.12 + 3$
$f(1) = 2.88$

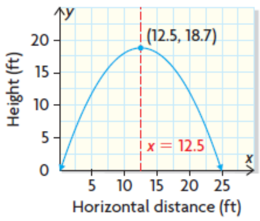
x	0	1	2	12	13
f(x)	0	2.88	5.52	18.72	18.72

I continued to increase x by intervals of 1 until I noticed a repeat in my values. A height of 18.72 ft occurs at horizontal distances of 12 ft and 13 ft.

Based on symmetry and the table of values, the maximum value of $f(x)$ will occur halfway between (12, 18.72) and (13, 18.72).

The arch of water will reach a maximum height between 12 ft and 13 ft from the opening in the ground.

maximum value
The greatest value of the dependent variable in a relation.



I used my table of values to sketch the graph. I extended the graph to the x-axis. I knew that my sketch represented only part of the function, since I am only looking at the water when it is above the ground.

$$x = \frac{12 + 13}{2}$$
$x = 12.5$

I used two points with the same y-value, (12, 18.72) and (13, 18.72), to determine the equation of the axis of symmetry.

Equation of the axis of symmetry:
$x = 12.5$

Height at the vertex:

$$\begin{aligned} f(x) &= -0.12x^2 + 3x \\ f(12.5) &= -0.12(12.5)^2 + 3(12.5) \\ f(12.5) &= -0.12(156.25) + 37.5 \\ f(12.5) &= -18.75 + 37.5 \\ f(12.5) &= 18.75 \end{aligned}$$

I knew that the x-coordinate of the vertex is 12.5, so I substituted 12.5 into the equation to determine the height of the water at this horizontal distance.

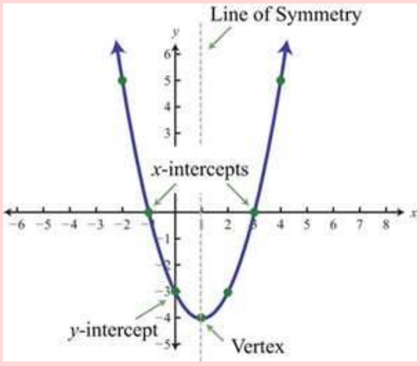
The water reaches a maximum height of 18.75 ft when it is 12.5 ft from the opening in the ground.

The water can reach a maximum horizontal distance of 25 ft from the opening in the ground.

Due to symmetry, the opening in the ground must be the same horizontal distance from the axis of symmetry as the point on the ground where the water lands. I simply multiplied the horizontal distance to the axis of symmetry by 2.

The domain of this function is $0 \le x \le 25$, where $x \in \mathbb{R}$.

Equation of axis of symmetry will be x = ?



Vertex is (x , y)

axis of symmetry

max / min y value

y-intercept - where graph crosses the y-axis (x = 0)
- this is the "c" value in $ax^2 + bx + c$

x-intercept(s) - where graph crosses the x-axis (y = 0)
- there can be 0, 1, or 2 x-intercepts

Domain - {x∈ ?}

Range {y∈ ?, y≤max. vertex or y≥min. vertex}