Some equations can be solved by factoring:

- start with writing the equation in standard form

$$y = ax^2 + bx + c$$

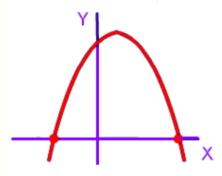
- set each factor equal to zero, and solve each linear equation

$$0 = a(x - r)(x - s)$$

$$x - r = 0 \quad \text{or} \quad x - s = 0$$

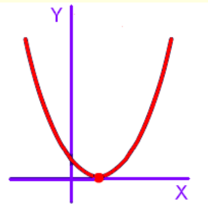
$$x = r \qquad\qquad x = s$$

- Each solution is a solution(root) to the original equation (or the x-intercepts of the graph)



two x - intercept

- If both factors are the same then there is only one solution(root) to the equation (or one x-intercept)



one x - intercept

Factoring Quadratics using "DECOMPOSITION" ($a \neq 1$)

To factor a "hard" quadratic, we have to handle all three coefficients, not just the two we handled in the "easy" case, because the leading coefficient (the number on the x^2 term) is not 1. The first step in factoring will be to multiply "a" and "c"; then we'll need to find factors of the product "ac" that add up to "b".

- Factor $2x^2 + x - 6$.

$$2x^2 + x - 6 = 2x^2 + 4x - 3x - 6 = 2x(x + 2) - 3(x + 2) = (x + 2)(2x - 3)$$

- Factor $4x^2 - 19x + 12$.

- Factor $2x^2 - 4x - 16$

EXAMPLE 3 Solving a quadratic equation with only one root

Solve and verify the following equation:

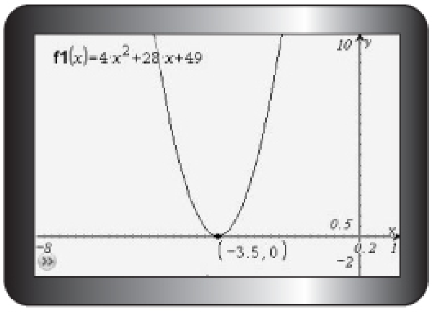
$$4x^2 + 28x + 49 = 0$$

Arya's Solution

$$4x^2 + 28x + 49 = 0$$
$$(2x + 7)(2x + 7) = 0$$

I factored the trinomial. I noticed that both factors are the same, so there is only one root.

$$2x + 7 = 0$$
$$x = -3.5$$



I decided to verify my solution by graphing the corresponding quadratic function.

I noticed that the vertex of the function is on the x-axis at -3.5, so my solution makes sense.

Factoring Quadratics using "DIFFERENCE OF SQUARES" (2 terms)

$$x^2 - 4 = (x \quad)(x \quad)$$

You need factors of -4 that add up to zero, so use -2 and $+2$:

$$x^2 - 4 = (x - 2)(x + 2)$$

- **Factor $x^2 - 16$**

  This is $x^2 - 4^2$, so I get:

  $$x^2 - 16 = x^2 - 4^2 = \mathbf{(x - 4)(x + 4)}$$

- **Factor $4x^2 - 25$**

  This is $(2x)^2 - 5^2$, so I get:

  $$4x^2 - 25 = (2x)^2 - 5^2 = \mathbf{(2x - 5)(2x + 5)}$$

$$x^4 - 1 = (x^2)^2 - 1^2 = (x^2 - 1)(x^2 + 1)$$

$$= ((x)^2 - (1)^2)(x^2 + 1)$$

$$= \mathbf{(x - 1)(x + 1)(x^2 + 1)}$$

APPLY the Math

EXAMPLE 2 Solving a quadratic equation using a difference of squares

Determine the roots of the following equation:

$$75p^2 - 192 = 0$$

Verify your solution.

Alberto's Solution

$$75p^2 - 192 = 0$$
$$\frac{75p^2}{3} - \frac{192}{3} = \frac{0}{3}$$

I noticed that 3 is a factor of both 75 and 192.

$$25p^2 - 64 = 0$$
$$(5p - 8)(5p + 8) = 0$$

I noticed that $25p^2$ and 64 are both perfect squares, so $25p^2 - 64$ is a difference of squares.

$$5p - 8 = 0 \quad \text{or} \quad 5p + 8 = 0$$
$$5p = 8 \qquad\qquad 5p = -8$$
$$p = \frac{8}{5} \qquad\qquad p = -\frac{8}{5}$$

I determined the roots.

The roots are $\frac{8}{5}$ and $-\frac{8}{5}$.

$$75p^2 - 192 = 0$$
$$75p^2 = 192$$
$$p^2 = \frac{192}{75}$$
$$p^2 = \frac{64}{25}$$
$$p = \pm\sqrt{\frac{64}{25}}$$
$$p = \pm\frac{8}{5}$$

I decided to verify my solutions by solving the equation using a different method.

I isolated p^2 and then took the square root of each side. I knew that p^2 has two possible square roots, one positive and the other negative.

My solution matched the solution I obtained by factoring.

What Happened When the Boarding House Blew Up?

Factor each trinomial below. Find one of the factors in **each** column of binomials. Notice the letter next to one factor and the number next to the other. Write the letter in the box at the bottom of the page that contains the matching number.

1. $3x^2 + 7x + 2$
2. $2x^2 + 5x + 3$
3. $3x^2 - 16x + 5$
4. $7x^2 - 9x + 2$
5. $6u^2 + 5u + 1$
6. $8u^2 - 9u + 1$
7. $10u^2 + 17u + 3$
8. $9u^2 - 9u + 2$
9. $5u^2 + 11u + 6$

5 $(5u + 3)$	Y $(3u - 2)$
3 $(x - 1)$	E $(x - 5)$
8 $(3x + 1)$	G $(8u - 1)$
14 $(3u - 1)$	O $(7x - 2)$
6 $(2u + 3)$	R $(5u + 1)$
15 $(x + 1)$	W $(x + 2)$
9 $(5u + 6)$	L $(7x + 2)$
7 $(2u + 1)$	I $(2x + 3)$
11 $(3x - 1)$	E $(u + 1)$
17 $(u - 1)$	S $(3u + 1)$

10. $3n^2 + 2n - 1$
11. $5n^2 - 4n - 1$
12. $2n^2 + 5n - 3$
13. $7n^2 - 13n - 2$
14. $3t^2 + 14t - 5$
15. $4t^2 - 11t + 7$
16. $6t^2 + 5t - 1$
17. $3t^2 - 20t - 7$

12 $(3t - 1)$	N $(n + 3)$
5 $(n - 1)$	R $(t - 1)$
4 $(3t + 1)$	P $(2t + 1)$
10 $(n - 2)$	O $(n + 1)$
13 $(t + 1)$	F $(t + 5)$
2 $(3n - 1)$	E $(5n + 1)$
16 $(2n - 1)$	M $(t - 7)$
4 $(3t - 7)$	R $(7n + 1)$
1 $(4t - 7)$	L $(6t - 1)$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17

OBJECTIVE 3–o: To factor trinomials of the form $ax^2 + bx + c$, where a is a positive integer greater than 1.

How Can Fishermen Save Gas ?

Factor each trinomial below. Find one of the factors in **each** column of binomials. Notice the letter next to one factor and the number next to the other. Write the letter in the box at the bottom of the page that contains the matching number.

Trinomial		
1 $4n^2 - 49$	3 $(n+1)$	O $(n-3)$
2 $n^2 + 8n + 12$	11 $(n+2)$	G $(2n-7)$
3 $n^2 - 9n + 20$	2 $(n+8)$	P $(n-5)$
4 $n^2 + 16n + 64$	9 $(2n+7)$	S $(3n-5)$
5 $n^2 + 2n - 15$	4 $(n+5)$	Y $(n+8)$
6 $3n^2 - 8n + 5$	18 $(n-1)$	K $(3n-1)$
	14 $(n-4)$	A $(n+6)$

Trinomial		
7 $a^2 + 4a - 21$	1 $(a-5)$	G $(2a+1)$
8 $5a^2 + 9a - 2$	13 $(a+7)$	B $(a-6)$
9 $2a^2 + 11a + 15$	5 $(5a+1)$	P $(a-3)$
10 $1 - 9a^4$	7 $(a+2)$	O $(a+3)$
11 $a^2 - 11a + 30$	15 $(a-1)$	I $(5a-1)$
12 $10a^2 - 3a - 1$	8 $(1-3a^2)$	R $(2a-1)$
	16 $(2a+5)$	N $(1+3a^2)$

Trinomial		
13 $8u^2 + 19u + 6$	10 $(u+3)$	M $(u+1)$
14 $25u^2 - 20u + 4$	12 $(2u+9)$	B $(2u+1)$
15 $3u^2 - 11u - 14$	17 $(u-3)$	O $(8u+3)$
16 $u^2 - 4u - 21$	3 $(5u-2)$	L $(2u-1)$
17 $6u^2 + 17u - 10$	6 $(3u-14)$	C $(u-7)$
18 $2u^2 + 5u - 18$	15 $(u+2)$	R $(u-2)$
	17 $(3u+10)$	F $(5u-2)$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18

OBJECTIVE 3–p: To factor trinomials using the methods on preceding pages (review).

FM11-7s1.gsp

7s2e2 final.mp4

fm7s2-p8.tns

FM11-7s3.gsp

fm7s3-p1.tns

FM11-7s3-2.gsp

fm7s3-p2.tns

fm7s3-p8.tns

FM11-7s4.gsp

7s4e3 final.mp4

fm7s4-p11.tns

7s5e2 finalt.mp4