# Besides Graphing to find roots, some equations can be solved by Factoring.

Correctly solving the equation:

$$4x^2 - 9x = 0$$
$$x(4x - 9) = 0$$

To solve the equation, I rewrote it in standard form and then factored the left side.

$$x = 0 \quad \text{or} \quad 4x - 9 = 0$$
$$x = 0 \quad \text{or} \quad 4x = 9$$
$$x = 2.25$$

For my equation to be true, either $x$ or $4x - 9$ must equal 0.

Verify:

$$4x^2 - 9x = 0$$

$x = 0$

| LS | RS |
|---|---|
| $4x^2 - 9x$ | $0$ |
| $4(0)^2 - 9(0)$ | |
| $0 - 0$ | |
| $0$ | |

LS = RS

$x = 2.25$

| LS | RS |
|---|---|
| $4x^2 - 9x$ | $0$ |
| $4(2.25)^2 - 9(2.25)$ | |
| $20.25 - 20.25$ | |
| $0$ | |

LS = RS

I verified each solution by substituting it into the original equation. For both solutions, the left side is equal to the right side. Therefore, both solutions are correct.

# Factoring "EASY" Quadratics (a = 1)

- **Factor $x^2 + 5x + 6$.**

  I need to find factors of 6 that add up to 5. Since 6 can be written as the product of 2 and 3, and since $2 + 3 = 5$, then I'll use 2 and 3. I know from multiplying polynomials that this quadratic is formed from multiplying two factors of the form "$(x + m)(x + n)$", for some numbers $m$ and $n$. So I'll draw my parentheses, with an "$x$" in the front of each:

  $(x \quad )(x \quad )$

  Then I'll write in the two numbers that I found above:

  $(x + 2)(x + 3)$

  This is the answer: $x^2 + 5x + 6 = \mathbf{(x + 2)(x + 3)}$

This is how all of the "easy" quadratics will work: you will find factors of the constant term that add up to the middle term, and use these factors to fill in your parentheses.

- **Factor $x^2 - 5x + 6$.**

  The constant term is 6, but the middle coefficient this time is negative. Since I multiplied to a positive six, then the factors must have the same sign. (Remember that two negatives multiply to a positive.) Since I'm adding to a negative ($-5$), then both factors must be negative. So rather than using 2 and 3, as in the first example, this time I will use $-2$ and $-3$:

  $x^2 - 5x + 6 = \mathbf{(x - 2)(x - 3)}$

- **Factor $x^2 - x - 6$.**

  This looks just like the previous case, except that now the middle term is negative. I still want factors with opposite signs, and I still want factors that are one apart, but this time the larger factor gets the "minus" sign:

  $x^2 - x - 6 = \mathbf{(x - 3)(x + 2)}$

## LEARN ABOUT the Math

The entry to the main exhibit hall in an art gallery is a parabolic arch. The arch can be modelled by the function

$$h(w) = -0.625w^2 + 5w$$

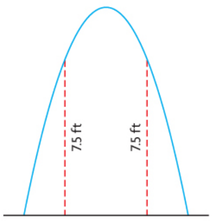
where the height, $h(w)$, and width, $w$, are measured in feet. Several sculptures are going to be delivered to the exhibit hall in crates. Each crate is a square-based rectangular prism that is 7.5 ft high, including the wheels. The crates must be handled as shown, to avoid damaging the fragile contents.


**?** What is the maximum width of a 7.5 ft high crate that can enter the exhibit hall through the arch?

### EXAMPLE 1 Solving a quadratic equation by factoring

Determine the distance between the two points on the arch that are 7.5 ft high.

**Brooke's Solution**




I sketched the situation.

The crate can only fit through the part of the arch that is at least 7.5 ft high. The arch is exactly 7.5 ft high at two points.

The following function describes the arch:
$h(w) = -0.625w^2 + 5w$

The height of the crate is 7.5 ft.

$$7.5 = -0.625w^2 + 5w$$

I wrote an equation, substituting 7.5 for $h(w)$.

$$0.625w^2 - 5w + 7.5 = 0$$

I rewrote the equation in standard form.

I decided to subtract $-0.625w^2 + 5w$ from both sides so the coefficient of $w^2$ would be positive.

$$\frac{0.625w^2}{0.625} - \frac{5w}{0.625} + \frac{7.5}{0.625} = \frac{0}{0.625}$$

I divided by 0.625 to simplify the equation.

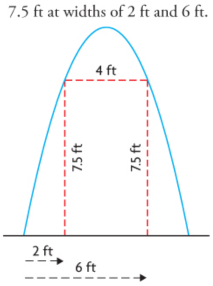
$$w^2 - 8w + 12 = 0$$
$$(w - 2)(w - 6) = 0$$

I factored the equation.

$$w - 2 = 0 \quad \text{or} \quad w - 6 = 0$$
$$w = 2 \qquad\qquad w = 6$$

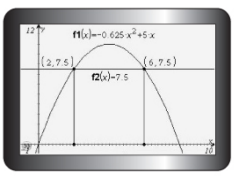
If the product of two factors is 0, then at least one factor must equal 0.

The parabola reaches a height of exactly 7.5 ft at widths of 2 ft and 6 ft.




I determined the difference between the widths to determine the maximum width of the crate.

To fit through the archway, the crate cannot be more than 4 ft wide.




I checked my solution by graphing $y = -0.625x^2 + 5x$ and $y = 7.5$.

The $x$-coordinates of the points of intersection are 2 and 6, so my solution is correct.

# Factoring "PERFECT SQUARE" Quadratics

- **Write $16x^2 - 48x + 36$ as a squared binomial.**

  The first term, $16x^2$, is the square of $4x$, and the last term, $36$, is the square of $6$. Actually, since the middle term has a "minus" sign, the $36$ is the square of $-6$. Just to be sure, I'll make sure that the middle term matches the pattern: $(4x)(-6)(2) = -48x$. It's a match, so this is a perfect square:

  $$16x^2 - 48x + 36 = \mathbf{(4x - 6)^2}$$

- **Is $4x^2 - 25x + 36$ a perfect square trinomial?**

  The first term, $4x^2$, is the square of $2x$, and the last term, $36$, is the square of $6$ (or, in this case, $-6$, if this is a perfect square). Checking the middle term, I get $(2x)(-6)(2) = -24x$, which does not match the middle term. So:

  **this is *not* a perfect square trinomial.**

That's all there is to perfect squares.

# When Is a Wrestler "King of the Ring"?

Factor each trinomial below. Find your answer and notice the letter next to it. Write this letter in the box containing the number of that exercise. Keep working and you will get the gripping answer to the title question.

1. $n^2 + 6n + 5$
2. $n^2 + 7n + 10$
3. $n^2 - 7n + 12$
4. $n^2 - 11n + 28$
5. $n^2 + 2n - 15$
6. $n^2 - 5n - 24$
7. $n^2 + n - 56$

Answers:

(L) $(n + 2)(n + 6)$
(H) $(n + 5)(n - 3)$
(W) $(n + 5)(n + 1)$
(E) $(n - 3)(n - 4)$
(B) $(n - 1)(n + 15)$
(S) $(n + 8)(n - 7)$
(H) $(n + 2)(n + 5)$
(E) $(n - 8)(n + 3)$
(R) $(n - 12)(n - 2)$
(N) $(n - 7)(n - 4)$

8. $t^2 + 10t + 16$
9. $t^2 - 15t + 50$
10. $t^2 + 8t - 9$
11. $t^2 - 7t - 30$
12. $t^2 - t - 30$
13. $t^2 + 14t + 48$
14. $t^2 + 8t - 48$

Answers:

(N) $(t - 6)(t + 5)$
(V) $(t - 25)(t + 2)$
(T) $(t - 5)(t - 10)$
(T) $(t + 6)(t + 8)$
(O) $(t - 10)(t + 3)$
(B) $(t + 15)(t - 2)$
(I) $(t + 8)(t + 2)$
(H) $(t - 4)(t + 12)$
(S) $(t + 9)(t - 1)$
(A) $(t - 24)(t + 2)$

15. $a^2 + 5ab + 6b^2$
16. $a^2 - 4ab - 21b^2$
17. $a^2 + 6ab - 7b^2$
18. $a^2 - 14ab - 32b^2$
19. $a^2 - 29ab + 100b^2$
20. $a^2 + 7ab - 18b^2$
21. $a^2 + 2ab + b^2$

Answers:

(K) $(a - 8b)(a + 4b)$
(H) $(a + 7b)(a - b)$
(A) $(a - 20b)(a + 5b)$
(E) $(a + 2b)(a + 3b)$
(W) $(a + 9b)(a - 2b)$
(T) $(a - 7b)(a + 3b)$
(O) $(a - 25b)(a - 4b)$
(S) $(a + 6b)(a + 3b)$
(N) $(a + b)(a + b)$
(R) $(a - 16b)(a + 2b)$

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
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OBJECTIVE 3–n: To factor trinomials of the form $x^2 + bx + c$, where $c$ is positive or negative (review).

# What Happens If the Jolly Green Giant Steps on Your House?

For exercises in the first column, express each square as a trinomial. For the remaining exercises, factor each trinomial as the square of a binomial, if possible. (If this is not possible, the correct answer is "not possible.") Find your answer below. Write the letter of the exercise in the box containing the number of its answer.

**Express as a trinomial:**

- (E) $(u+3)^2$
- (O) $(u-8)^2$
- (S) $(2u+5)^2$
- (L) $(1-4u)^2$
- (T) $(u+2v)^2$
- (U) $(7u-3v)^2$
- (O) $(uv+6)^2$

Answers:

- (13) $4u^2+20u+25$
- (3) $4u^2+16u+25$
- (9) $u^2+6u+9$
- (10) $u^2+4uv+4v^2$
- (14) $49u^2-31uv+9v^2$
- (6) $1-8u+16u^2$
- (2) $u^2-16u+64$
- (18) $u^2v^2+12uv+36$
- (5) $u^2+7uv+4v^2$
- (12) $49u^2-42uv+9v^2$

**Factor:**

- (E) $t^2+4t+4$
- (U) $t^2-12t+36$
- (L) $t^2-18t+81$
- (Y) $25+10t+t^2$
- (W) $4t^2+20t+25$
- (S) $9t^2-12t+4$
- (I) $t^2+10t+20$

Answers:

- (5) not possible
- (7) $(t-9)^2$
- (19) $(t-12)^2$
- (4) $(2t+5)^2$
- (15) $(t+2)^2$
- (21) $(3t-2)^2$
- (16) $(2t-9)^2$
- (3) $(t-6)^2$
- (1) $(5+t)^2$
- (8) $(3t-5)^2$

**Factor:**

- (D) $49a^2+14a+1$
- (O) $16a^2-24a+9$
- (G) $a^2-8a+64$
- (M) $a^2+2ab+b^2$
- (H) $a^2+10ab+25b^2$
- (R) $4a^2-12ab+9b^2$
- (M) $100a^2-20ab+b^2$

Answers:

- (8) not possible
- (11) $(10a-3b)^2$
- (16) $(7a+1)^2$
- (11) $(10a-b)^2$
- (20) $(a+b)^2$
- (17) $(2a-3b)^2$
- (19) $(4a-3)^2$
- (20) $(a+3b)^2$
- (14) $(a+5b)^2$
- (19) $(4a-8)^2$

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
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  OBJECTIVE 3–j: To find squares of binomials; to factor trinomial squares.

Attachments

FM11-7s1.gsp

7s2e2 final.mp4

fm7s2-p8.tns

FM11-7s3.gsp

fm7s3-p1.tns

FM11-7s3-2.gsp

fm7s3-p2.tns

fm7s3-p8.tns

FM11-7s4.gsp

7s4e3 final.mp4

fm7s4-p11.tns

7s5e2 finalt.mp4