Solving Quadratic Equations by Partial Factoring

Sketch the graph of the following quadratic function:

$$f(x) = -x^2 + 6x + 10$$

State the domain and range of the function.

$f(x) = -x^2 + 6x + 10$
$f(x) = -x(x - 6) + 10$

I couldn't identify two integers with a product of 10 and a sum of 6, so I couldn't factor the expression. I decided to remove a partial factor of $-x$ from the first two terms. I did this so that I could determine the x-coordinates of the points that have 10 as their y-coordinate.

$-x = 0$, $x = 0$, $f(0) = 10$
$x - 6 = 0$, $x = 6$, $f(6) = 10$

The points (0, 10) and (6, 10) belong to the given quadratic function.

I determined two points in the function by setting each partial factor equal to zero.

When either factor is zero, the product of the factors is zero, so the value of the function is 10.

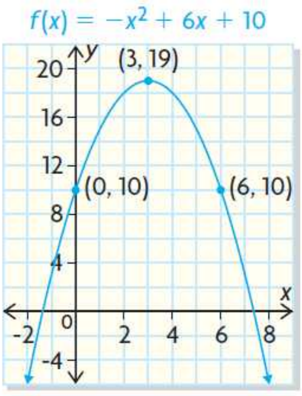
$$x = \frac{0 + 6}{2}$$
$x = 3$

Because (0, 10) and (6, 10) have the same y-coordinate, they are the same horizontal distance from the axis of symmetry. I determined the equation of the axis of symmetry by calculating the mean of the x-coordinates of these two points.

$f(3) = -(3)^2 + 6(3) + 10$
$f(3) = -9 + 18 + 10$
$f(3) = 19$

I determined the y-coordinate of the vertex.

The vertex is (3, 19).



The coefficient of the x^2 term is negative, so the parabola opens downward.

I used the vertex, as well as (0, 10) and (6, 10), to sketch the parabola.

Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y \leq 19, y \in R\}$

The only restriction on the variables is that y must be less than or equal to 19, the maximum value of the function.

EXAMPLE 4 Solving a problem modelled by a quadratic function in factored form

The members of a Ukrainian church hold a fundraiser every Friday night in the summer. They usually charge \$6 for a plate of perogies. They know, from previous Fridays, that 120 plates of perogies can be sold at the \$6 price but, for each \$1 price increase, 10 fewer plates will be sold. What should the members charge if they want to raise as much money as they can for the church?

Krystina's Solution: Using the properties of the function

Let y represent the total revenue.
$y =$ (Number of plates)(Price)
Let x represent the number of \$1 price increases.
$y = (120 - 10x)(6 + x)$

> For each price increase, x, I knew that $10x$ fewer plates will be sold.
>
> If I expanded the factors in my function, I would create an x^2 term. This means that the function I have defined is quadratic and its graph is a parabola.

$0 = (120 - 10x)(6 + x)$
$120 - 10x = 0$ or $6 + x = 0$
$-10x = -120$ $\quad x = -6$
$x = 12$
The x-intercepts are $x = -6$ and $x = 12$.

> To determine the zeros of the function, I substituted zero for y. A product is zero only when one or both of its factors are zero, so I set each factor equal to zero and solved each equation.

$$x = \frac{12 + (-6)}{2}$$
$x = 3$

$y = (120 - 10x)(6 + x)$
$y = [120 - 10(3)][6 + (3)]$
$y = (90)(9)$
$y = 810$
The coordinates of the vertex are (3, 810).

> I determined the equation of the axis of symmetry for the parabola by calculating the mean distance between the x-intercepts.
>
> I determined the y-coordinate of the vertex by substituting into my initial equation.

To generate as much revenue as possible, the members of the church should charge \$6 + \$3 or \$9 for a plate of perogies. This will provide revenue of \$810.

> The vertex describes the maximum value of the function. Maximum sales of \$810 occur when the price is raised by \$3.

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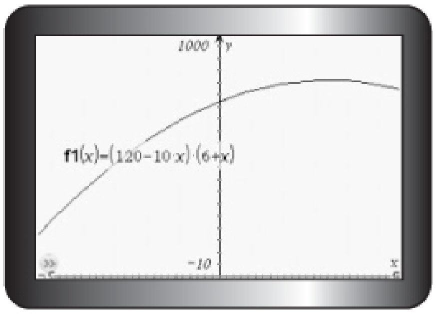
Jennifer's Solution: Using graphing technology

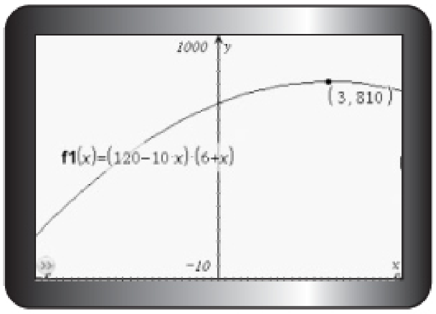
Revenue = (Number of plates)(Price)

$$y = (120 - 10x)(6 + x)$$

I let y represent Revenue and I let x represent the number of \$1 price increases. For each \$1 price increase, I knew that 10 fewer plates will be sold.



I graphed the equation on a calculator. Since a reduced price may result in maximum revenue, I set my domain to a minimum value of −5 and a maximum value of 5.



I used the calculator to locate the vertex of the parabola.

The members of the church should charge \$3 more than the current price of \$6 for a plate of perogies. If they charge \$9, they will reach the maximum revenue of \$810.

Attachments

FM11-7s1.gsp

7s2e2 final.mp4

fm7s2-p8.tns

FM11-7s3.gsp

fm7s3-p1.tns

FM11-7s3-2.gsp

fm7s3-p2.tns

fm7s3-p8.tns

FM11-7s4.gsp

7s4e3 final.mp4

fm7s4-p11.tns