To SOLVE a quadratic equation in the form $y = ax^2 + bx + c$

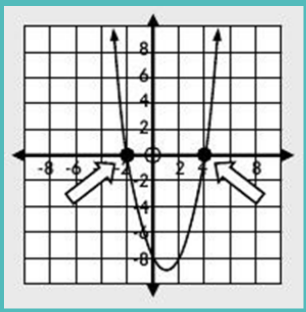
means to find its roots which is the same as

the zeros of the function $f(x) = ax^2 + bx + c$

$0 = ax^2 + bx + c$

which is the same as

the x-intercepts of a quadratic graph.



**Remember that x-intercepts are those value(s) of x when y = 0.
(Remember there can be 0, 1, or 2 x - intercepts)

1.4 ROOTS OF QUADRATIC EQUATIONS

standard

A quadratic equation of the form $y = ax^2 + bx + c$ is said to be in [redacted] form. If we wish to *solve* the equation, however, we would have to examine $ax^2 + bx + c = 0$ to determine the values of x that result in $y = 0$. The values that satisfy this relationship are the solutions or *roots* or *zeros* of the equation. You already know of several methods in which to solve a quadratic equation which shall be reviewed here. We will also introduce you to other methods which will be new to you.

The ***x-intercepts*** of a function are also referred to as the ***roots*** or ***zeros*** of the function.

Let us examine the various methods we can use to determine the roots of a quadratic equation.

A. GRAPHING

If the quadratic function is given in the form $y = ax^2 + bx + c$, the x-intercepts can be determined by letting $y = 0$ and thus solving the equation $ax^2 + bx + c = 0$.

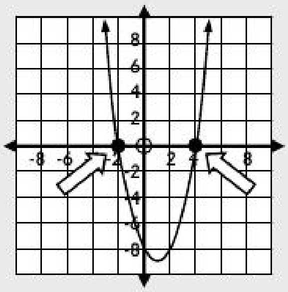
EXAMPLE 19

Solve each quadratic equation by graphing.

a. $x^2 - 2x - 8 = 0$ b. $x^2 + 4x + 4 = 0$

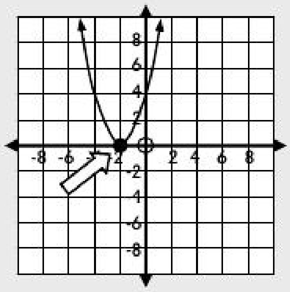
Solution

a. Sketch the graph of $y = x^2 - 2x - 8$



The graph crosses the x-axis at -2 and 4.
∴ the solutions are x = -2 and x = 4.

b. Sketch the graph of $y = x^2 + 4x + 4$



The graph crosses the x-axis at -2.
∴ the solution is x = -2.

FINDING ROOTS

- You can still find the roots if you do not rearrange the quadratic equation into std form ($y = ax^2 + bx + c$)

- You must graph both the left side and the right side of the equation.

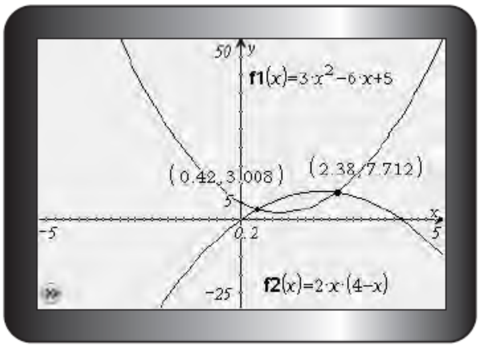
- Where these graphs intersect will <u>not</u> be the root(s), however, the x value(s) of the intersection point(s) will be the roots.

Determine the roots of this quadratic equation. Verify your answers.

$$3x^2 - 6x + 5 = 2x(4 - x)$$

$f(x) = 3x^2 - 6x + 5$

$g(x) = 2x(4 - x)$



The solutions are $x = 0.420$ and $x = 2.380$.

Verify:

$3x^2 - 6x + 5 = 2x(4 - x)$

$x = 0.420$

LS	RS
$3(0.420)^2 - 6(0.420) + 5$	$2(0.420)(4 - 0.420)$
$3.009 \ldots$	$3.007 \ldots$

LS $\doteq$ RS

Verify:

$3x^2 - 6x + 5 = 2x(4 - x)$

$x = 2.380$

LS	RS
$3(2.380)^2 - 6(2.380) + 5$	$2(2.380)(4 - 2.380)$
$7.713 \ldots$	$7.711 \ldots$

LS $\doteq$ RS

The roots are $x = 0.420$ and $x = 2.380$.

B. FACTORING

Factoring can also be used to determine the roots of a quadratic equation. A useful property when using this method is known as the "Zero Product Property", which states that if the product of two numbers is zero, then at least one of them must be zero.

Zero Product Property

If $a \times b = 0$, then $a = 0$ or $b = 0$, or both, where a and b are real numbers.

EXAMPLE 20

Determine the roots of each quadratic equation.

a. $x^2 - 10x + 16 = 0$ b. $2x^2 - 5x - 3 = 0$ c. $x^2 + 3x = 10$

Solution

a.
$$x^2 - 10x + 16 = 0$$
$$(x - 8)(x - 2) = 0$$
$$x - 8 = 0 \quad \text{or} \quad x - 2 = 0$$
$$x = 8 \quad \text{or} \quad x = 2$$

b.
$$2x^2 - 5x - 3 = 0$$
$$(2x + 1)(x - 3) = 0$$
$$2x + 1 = 0 \quad \text{or} \quad x - 3 = 0$$
$$2x = -1 \quad \text{or} \quad x = 3$$
$$x = -\frac{1}{2}$$

c.
$$x^2 + 3x = 10$$
$$x^2 + 3x - 10 = 0$$
$$(x - 2)(x + 5) = 0$$
$$x - 2 = 0 \quad \text{or} \quad x + 5 = 0$$
$$x = 2 \quad \text{or} \quad x = -5$$

- You can then let x = 0, to find the y-intercept.
- You can find the midpoint of the roots to determine the axis of symmetry.
- You can substitute this x value(axis of symmetry) into the equation to find the corresponding y-value, which is the max/min value and the y-coordinate of the vertex.
- You now have enough information to sketch the graph!

Attachments

FM11-7s1.gsp

7s2e2 final.mp4

fm7s2-p8.tns