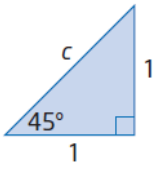
Special Right Triangles

For angles of 30°, 45°, and 60°, you can determine the **exact values** of trigonometric ratios.

exact value

- answers involving radicals are exact, unlike approximated decimal values
- fractions such as $\frac{1}{3}$ are exact, but an approximation of $\frac{1}{3}$ such as 0.333 is not

Drawing the diagonal of a square with a side length of 1 unit gives a 45°-45°-90° triangle. This is an isosceles right triangle.



Use the Pythagorean Theorem to find the length of the hypotenuse.

$c^2 = a^2 + b^2$
$c^2 = 1^2 + 1^2$
$c^2 = 2$
$c = \sqrt{2}$

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\sin 45° = \frac{1}{\sqrt{2}} \qquad \cos 45° = \frac{1}{\sqrt{2}} \qquad \tan 45° = \frac{1}{1}$$

$$\tan 45° = 1$$

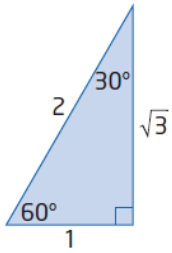
What are the three primary trigonometric ratios for the other acute angle in this triangle?

Drawing the altitude of an equilateral triangle with a side length of 2 units gives a 30°-60°-90° triangle.

Using the Pythagorean Theorem, the length of the altitude is $\sqrt{3}$ units.

$$\sin 60° = \frac{\sqrt{3}}{2} \qquad \cos 60° = \frac{1}{2} \qquad \tan 60° = \frac{\sqrt{3}}{1} = \sqrt{3}$$

$$\sin 30° = \frac{1}{2} \qquad \cos 30° = \frac{\sqrt{3}}{2} \qquad \tan 30° = \frac{1}{\sqrt{3}}$$



Which trigonometric ratios for 30° have exact decimal values? Which are irrational numbers?

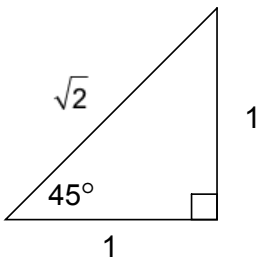
http://www.onlinemathlearning.com/reference-angles.html

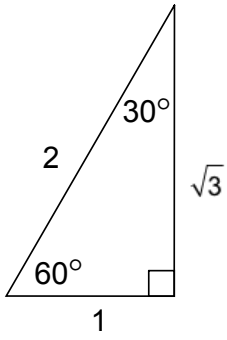
Chapter 2

Special Right Triangles

Write the exact values of trigonometric ratios for the special right triangles.

Answer





θ	sin θ	cos θ	tan θ
30°			
45°			
60°			

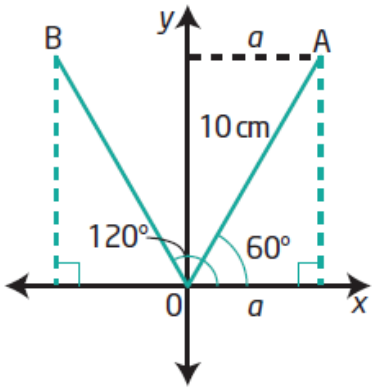
Example 4

Find an Exact Distance

Allie is learning to play the piano. Her teacher uses a metronome to help her keep time. The pendulum arm of the metronome is 10 cm long. For one particular tempo, the setting results in the arm moving back and forth from a start position of 60° to 120°. What horizontal distance does the tip of the arm move in one beat? Give an exact answer.

Solution

Draw a diagram to model the information.



OA represents the start position and OB the end position of the metronome arm for one beat. The tip of the arm moves a horizontal distance equal to a to reach the vertical position.

Find the horizontal distance a:

$$\cos 60° = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\frac{1}{2} = \frac{a}{10}$$

$$10\left(\frac{1}{2}\right) = a$$

$$5 = a$$

Why is $\frac{1}{2}$ substituted for cos 60°?

Because the reference angle for 120° is 60°, the tip moves the same horizontal distance past the vertical position to reach B.

The exact horizontal distance travelled by the tip of the arm in one beat is 2(5) or 10 cm.