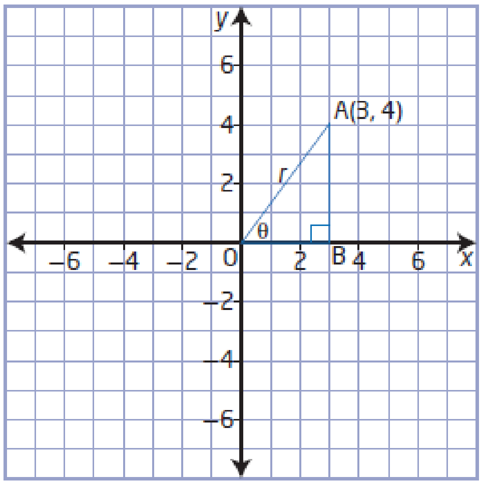
Investigate Trigonometric Ratios for Angles Greater Than 90°

Materials

- grid paper
- protractor

1. On grid paper, draw a set of coordinate axes.
   a) Plot the point A(3, 4). In which quadrant does the point A lie?
   b) Draw the angle in standard position with terminal arm passing through point A.

2. Draw a line perpendicular to the x-axis through point A. Label the intersection of this line and the x-axis as point B. This point is on the initial arm of ∠AOB.



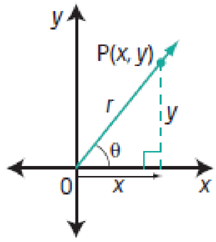
   a) Use the Pythagorean Theorem to determine the length of the hypotenuse, r.
   b) Write the primary trigonometric ratios for θ.
   c) Determine the measure of θ, to the nearest degree.

3. How is each primary trigonometric ratio related to the coordinates of point A and the radius r?

Link the Ideas

Finding the Trigonometric Ratios of Any Angle θ, where 0° ≤ θ < 360°

Suppose θ is any angle in standard position, and P(*x*, *y*) is any point on its terminal arm, at a distance *r* from the origin. Then, by the Pythagorean Theorem, $r = \sqrt{x^2 + y^2}$.



You can use a reference triangle to determine the three primary trigonometric ratios in terms of *x*, *y*, and *r*.

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

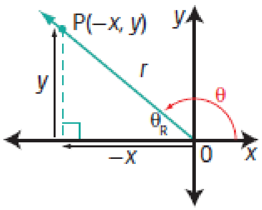
$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

The chart below summarizes the signs of the trigonometric ratios in each quadrant. In each, the horizontal and vertical lengths are considered as directed distances.

Quadrant II
$90° < \theta < 180°$

$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{-x}{r}$ $\tan\theta = \frac{y}{-x}$

$\sin\theta > 0$ $\cos\theta < 0$ $\tan\theta < 0$

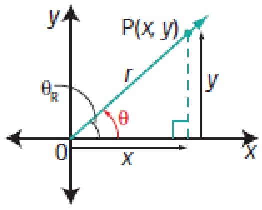


$\theta = 180° - \theta_R$

Quadrant I
$0° < \theta < 90°$

$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

$\sin\theta > 0$ $\cos\theta > 0$ $\tan\theta > 0$



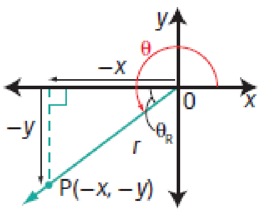
$\theta = \theta_R$

Why is *r* always positive?

Quadrant III
$180° < \theta < 270°$

$\sin\theta = \frac{-y}{r}$ $\cos\theta = \frac{-x}{r}$ $\tan\theta = \frac{-y}{-x}$

$\sin\theta < 0$ $\cos\theta < 0$ $\tan\theta > 0$

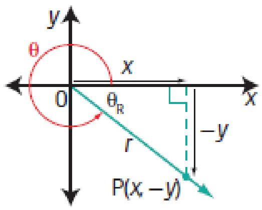


$\theta = 180° + \theta_R$

Quadrant IV
$270° < \theta < 360°$

$\sin\theta = \frac{-y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{-y}{x}$

$\sin\theta < 0$ $\cos\theta > 0$ $\tan\theta < 0$



$\theta = 360° - \theta_R$

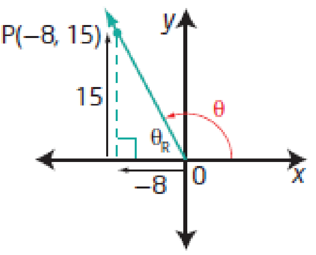
Example 1

Write Trigonometric Ratios for Angles in Any Quadrant

The point P(−8, 15) lies on the terminal arm of an angle, θ, in standard position. Determine the exact trigonometric ratios for sin θ, cos θ, and tan θ.

Solution

Sketch the reference triangle by drawing a line perpendicular to the *x*-axis through the point (−8, 15). The point P(−8, 15) is in quadrant II, so the terminal arm is in quadrant II.



Use the Pythagorean Theorem to determine the distance, *r*, from P(−8, 15) to the origin, (0, 0).

$r = \sqrt{x^2 + y^2}$

$r = \sqrt{(-8)^2 + (15)^2}$

$r = \sqrt{289}$

$r = 17$

The trigonometric ratios for θ can be written as follows:

$\sin\theta = \frac{y}{r}$ $\quad \cos\theta = \frac{x}{r}$ $\quad \tan\theta = \frac{y}{x}$

$\sin\theta = \frac{15}{17}$ $\quad \cos\theta = \frac{-8}{17}$ $\quad \tan\theta = \frac{15}{-8}$

$\cos\theta = -\frac{8}{17}$ $\quad \tan\theta = -\frac{15}{8}$

Example 2

Determine the Exact Value of a Trigonometric Ratio

Determine the exact value of cos 135°.

Solution

The terminal arm of 135° lies in quadrant II.
The reference angle is 180° − 135°, or 45°.
The cosine ratio is negative in quadrant II.

$$\cos 135° = -\frac{1}{\sqrt{2}}$$

Why are side lengths 1, 1, and $\sqrt{2}$ used?