Chapter 3 Quadratic Functions

Section 3.1 Investigating Quadratic Functions in Vertex Form

Section 3.1 Page 157 Question 1

a) The graph of $f(x) = 7x^2$ will open upward and be narrower than the graph of $f(x) = x^2$, since $a > 1$. The parabola will have a minimum value and a range of $\{y \mid y \geq 0, y \in \mathrm{R}\}$.

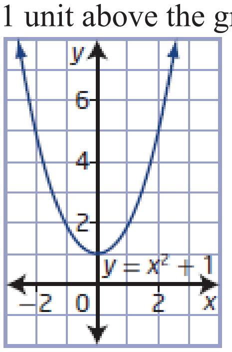
b) The graph of $f(x) = \frac{1}{6}x^2$ will open upward and be wider than the graph of $f(x) = x^2$, since $0 < a < 1$. The parabola will have a minimum value and a range of $\{y \mid y \geq 0, y \in \mathrm{R}\}$.

c) The graph of $f(x) = -4x^2$ will open downward and be narrower than the graph of $f(x) = x^2$, since $a < -1$. The parabola will have a maximum value and a range of $\{y \mid y \leq 0, y \in \mathrm{R}\}$.

d) The graph of $f(x) = -0.2x^2$ will open downward and be wider than the graph of $f(x) = x^2$, since $-1 < a < 0$. The parabola will have a maximum value and a range of $\{y \mid y \leq 0, y \in \mathrm{R}\}$.

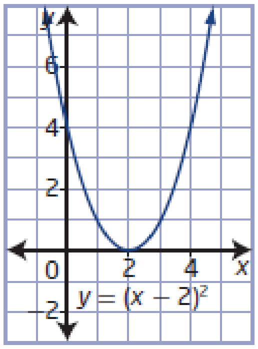
Section 3.1 Page 157 Question 2

a) $y = x^2$ and $y = x^2 + 1$
The shapes of the graphs are the same. Since $q = 1$ for $y = x^2 + 1$, its graph is translated 1 unit above the graph of $y = x^2$.



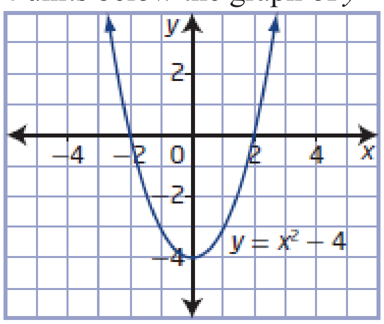
vertex: (0, 1)
axis of symmetry: $x = 0$
domain: $\{x \mid x \in \mathrm{R}\}$
range: $\{y \mid y \geq 1, y \in \mathrm{R}\}$
x-intercepts: none
y-intercept: 1

b) $y = x^2$ and $y = (x - 2)^2$
The shapes of the graphs are the same. Since $p = 2$ for $y = (x - 2)^2$, its graph is translated 2 units to the right of the graph of $y = x^2$.



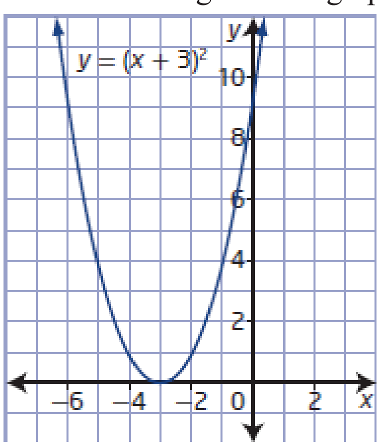
vertex: (2, 0)
axis of symmetry: $x = 2$
domain: $\{x \mid x \in \mathrm{R}\}$
range: $\{y \mid y \geq 0, y \in \mathrm{R}\}$
x-intercept: 2
y-intercept: 4

c) $y = x^2$ and $y = x^2 - 4$
The shapes of the graphs are the same. Since $q = -4$ for $y = x^2 - 4$, its graph is translated 4 units below the graph of $y = x^2$.



vertex: (0, –4)
axis of symmetry: $x = 0$
domain: $\{x \mid x \in \mathrm{R}\}$
range: $\{y \mid y \geq -4, y \in \mathrm{R}\}$
x-intercepts: –2 and 2
y-intercept: –4

d) $y = x^2$ and $y = (x + 3)^2$
The shapes of the graphs are the same. Since $p = -3$ for $y = (x + 3)^2$, its graph is translated 3 units to the right of the graph of $y = x^2$.



vertex: (–3, 0)
axis of symmetry: $x = -3$
domain: $\{x \mid x \in \mathrm{R}\}$
range: $\{y \mid y \geq 0, y \in \mathrm{R}\}$
x-intercept: –3
y-intercept: 9

Section 3.1 Page 157 Question 3

a) For $f(x) = (x + 5)^2 + 11$, $a = 1$, $p = -5$, and $q = 11$. Since $a = 1$, the shape of the graph is the same as the graph of $f(x) = x^2$. Since $p = -5$ and $q = 11$, the vertex is located at (–5, 11).

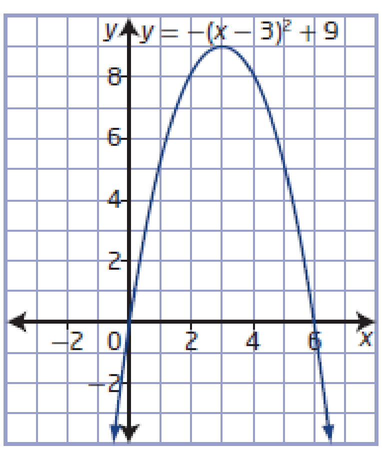
To sketch the graph of $f(x) = (x + 5)^2 + 11$, transform the graph of $f(x) = x^2$ by translating 5 units to the left and 11 units up.

b) For $f(x) = -3x^2 - 10$, $a = -3$, $p = 0$, and $q = -10$. Since $a < -1$, the shape of the graph is narrower than the graph of $f(x) = x^2$ and opens downward. Since $p = 0$ and $q = -10$, the vertex is located at (0, –10).
To sketch the graph of $f(x) = -3x^2 - 10$, transform the graph of $f(x) = x^2$ by
- multiplying the y-values by a factor of 3
- reflecting in the x-axis
- translating 10 units down

c) For $f(x) = 5(x + 20)^2 - 21$, $a = 5$, $p = -20$, and $q = -21$. Since $a > 1$, the shape of the graph is narrower than the graph of $f(x) = x^2$ and opens upward. Since $p = -20$ and $q = -21$, the vertex is located at (–20, –21).
To sketch the graph of $f(x) = 5(x + 20)^2 - 21$, transform the graph of $f(x) = x^2$ by
- multiplying the y-values by a factor of 5
- translating 20 units to the left and 21 units down

d) For $f(x) = -\frac{1}{8}(x - 5.6)^2 + 13.8$, $a = -\frac{1}{8}$, $p = 5.6$, and $q = 13.8$. Since $-1 < a < 0$, the shape of the graph is wider than the graph of $f(x) = x^2$ and opens downward. Since $p = 5.6$ and $q = 13.8$, the vertex is located at (5.6, 13.8).
To sketch the graph of $f(x) = -\frac{1}{8}(x - 5.6)^2 + 13.8$, transform the graph of $f(x) = x^2$ by
- multiplying the y-values by a factor of $\frac{1}{8}$
- reflecting in the x-axis
- translating 5.6 units to the right and 13.8 units up

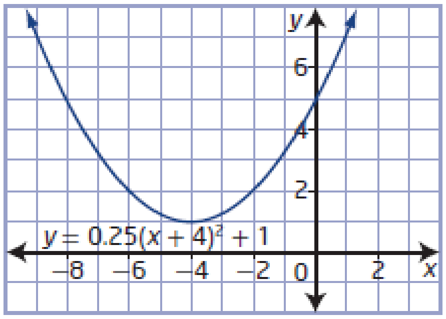
Section 3.1 Page 157 Question 4



a) For $y = -(x - 3)^2 + 9$, $a = -1$, $p = 3$, and $q = 9$.

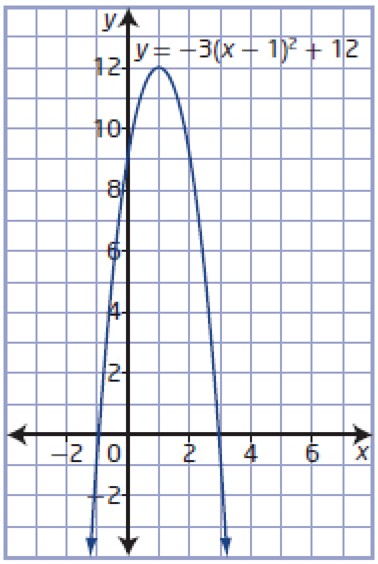
vertex: (3, 9)
axis of symmetry: $x = 3$
opens downward
maximum value of 9
domain: $\{x \mid x \in R\}$
range: $\{y \mid y \le 9, y \in R\}$
x-intercepts: 0 and 6
y-intercept: 0

b) For $y = 0.25(x + 4)^2 + 1$, $a = 0.25$, $p = -4$, and $q = 1$.



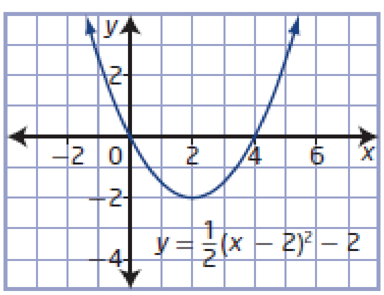
vertex: $(-4, 1)$
axis of symmetry: $x = -4$
opens upward
minimum value of 1
domain: $\{x \mid x \in \mathrm{R}\}$
range: $\{y \mid y \geq 1, y \in \mathrm{R}\}$
x-intercepts: none
y-intercept: 5

c) For $y = -3(x - 1)^2 + 12$, $a = -3$, $p = 1$, and $q = 12$.



vertex: $(1, 12)$
axis of symmetry: $x = 1$
opens downward
maximum value of 12
domain: $\{x \mid x \in \mathrm{R}\}$
range: $\{y \mid y \leq 12, y \in \mathrm{R}\}$
x-intercepts: -1 and 3
y-intercept: 9

d) For $y = \frac{1}{2}(x - 2)^2 - 2$, $a = \frac{1}{2}$, $p = 2$, and $q = -2$.



vertex: $(2, -2)$
axis of symmetry: $x = 2$
opens upward
minimum value of -2
domain: $\{x \mid x \in \mathrm{R}\}$
range: $\{y \mid y \geq -2, y \in \mathrm{R}\}$
x-intercepts: 0 and 4
y-intercept: 0