b) $\tan\theta_R = \dfrac{4}{9}$

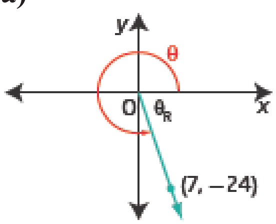
$\theta_R = \tan^{-1}\left(\dfrac{4}{9}\right)$

$\theta_R \approx 24°$

c) $\theta = 180° - 24°$
$\theta = 156°$, to the nearest degree

Section 2.2 Page 97 Question 13

a)



b) $\tan\theta_R = \dfrac{24}{7}$

$\theta_R = \tan^{-1}\left(\dfrac{24}{7}\right)$

$\theta_R \approx 74°$

c) $\theta = 360° - 74°$
$\theta = 286°$, to the nearest degree

Section 2.2 Page 97 Question 14

a) Given P(2, 4), then $x = 2$ and $y = 4$. Use the Pythagorean Theorem to determine r.
$r^2 = x^2 + y^2$
$r^2 = 2^2 + 4^2$
$r^2 = 20$
$r = \sqrt{20}$

Then, $\sin\theta = \dfrac{y}{r}$

$$\sin\theta = \frac{4}{\sqrt{20}} = \frac{4}{2\sqrt{5}} \text{ or } \frac{2\sqrt{5}}{5}$$

b) Given Q(4, 8), then $x = 4$ and $y = 8$.
Then, $r^2 = x^2 + y^2$
$r^2 = 4^2 + 8^2$
$r^2 = 80$
$r = \sqrt{80}$

$\sin\theta = \dfrac{y}{r}$

$$\sin\theta = \frac{8}{\sqrt{80}} = \frac{8}{4\sqrt{5}} \text{ or } \frac{2\sqrt{5}}{5}$$

c) Given R(8, 16), then $x = 8$ and $y = 16$.
Then, $r^2 = x^2 + y^2$
$r^2 = 8^2 + 16^2$

$$r^2 = 320$$
$$r = \sqrt{320}$$

$$\sin\theta = \frac{y}{r}$$

$$\sin\theta = \frac{16}{\sqrt{320}} = \frac{16}{8\sqrt{5}} \text{ or } \frac{2\sqrt{5}}{5}$$

d) The sine ratio is the same because the points P, Q, and R are all on the terminal arm of angle θ. The right triangles formed by *x*, *y*, and *r* in each case are all similar triangles so the ratio of corresponding sides are equal.

Section 2.2 Page 97 Question 15

a) Given P(k, 24) and $r = 25$,

$$\sin\theta = \frac{y}{r}$$

$$\sin\theta = \frac{24}{25}$$

$$\theta = \sin^{-1}\left(\frac{24}{25}\right)$$

$$\theta = 73.739...$$

The sine ratio is positive in quadrants I and II, so θ is approximately 74° or 180° − 74° = 106°.

b) Use the Pythagorean Theorem to determine *x*.

$$r^2 = x^2 + y^2$$
$$25^2 = x^2 + 24^2$$
$$x^2 = 625 - 576$$
$$x^2 = 49$$
$$x = \pm 7$$

Then, in quadrant I, $x = 7$, $y = 24$, and $r = 25$:

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$$

$$\sin\theta = \frac{24}{25} \qquad \cos\theta = \frac{7}{25} \qquad \tan\theta = \frac{24}{7}$$

Then, in quadrant II, $x = -7$, $y = 24$, and $r = 25$:

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$$

$$\sin\theta = \frac{24}{25} \qquad \cos\theta = \frac{-7}{25} = -\frac{7}{25} \qquad \tan\theta = \frac{24}{-7} = -\frac{24}{7}$$

Section 2.2 Page 97 Question 16

$\cos\theta = \dfrac{x}{r}$, so given $\cos\theta = \dfrac{1}{5}$, $x = 1$, and $r = 5$.

$\tan\theta = \dfrac{y}{x}$, so given $\tan\theta = 2\sqrt{6}$, $y = 2\sqrt{6}$.

Then, $\sin\theta = \dfrac{y}{r}$

$$\sin\theta = \frac{2\sqrt{6}}{5}$$

Section 2.2 Page 97 Question 17

At the equator, where the angle of dip is 0°, a point on the terminal arm is (1, 0).
So, $x = 1$, $y = 0$, and $r = 1$.

Then, $\sin\theta = \dfrac{y}{r}$ $\qquad \cos\theta = \dfrac{x}{r}$ $\qquad \tan\theta = \dfrac{y}{x}$

$\sin 0° = \dfrac{0}{1}$ or 0 $\qquad \cos 0° = \dfrac{1}{1}$ or 1 $\qquad \tan 0° = \dfrac{0}{1} = 0$

At the North and South Poles, where the angle of dip is 90°, a point on the terminal arm is (0, 1).
So, $x = 1$, $y = 0$, and $r = 1$.

Then, $\sin\theta = \dfrac{y}{r}$ $\qquad \cos\theta = \dfrac{x}{r}$ $\qquad \tan\theta = \dfrac{y}{x}$

$\sin 90° = \dfrac{1}{1}$ or 1 $\qquad \cos 90° = \dfrac{0}{1}$ or 0 $\qquad \tan 90° = \dfrac{1}{0}$ or undefined

Section 2.2 Page 98 Question 18

a) sin 151° = sin 29° is a true statement, because in quadrant I and II the sine ratio is positive and 29° is the reference angle for 151°.

b) cos 135° = sin 225° is a true statement, because in quadrant II the cosine ratio is negative, in quadrant III the sine ratio is negative, and 45° is the reference angle for both angles.

c) tan 135° = tan 225° is a false statement, because in quadrant II the tangent ratio is negative and in quadrant III the tangent ratio is positive.

d) sin 60° = cos 330° is a true statement, because using the special 30°-60°-90° reference triangle both have a value of $\dfrac{\sqrt{3}}{2}$.

e) sin 270° = cos 180° is a true statement, because both have a value of −1.

Section 2.2 Page 98 Question 19

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$
45°	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	undefined
120°	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$
135°	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	$-\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$	-1
150°	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}$ or $-\frac{\sqrt{3}}{3}$
180°	0	-1	0
210°	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$
225°	$-\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$	$-\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$	1
240°	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$
270°	-1	0	undefined
300°	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$
315°	$-\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	-1
330°	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}$ or $-\frac{\sqrt{3}}{3}$
360°	0	1	0

Section 2.2 Page 98 Question 20

a) The measure for $\angle A$ is 45°, for $\angle B$ is 135°, for $\angle C$ is 225°, and for $\angle D$ is 315°.

b) Use the special 45°-45°-90° reference triangle with sides $\frac{1}{\sqrt{2}}$, $\frac{1}{\sqrt{2}}$, and 1.

Then, A is on the terminal arm of 45°: $A\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$.

B is on the terminal arm of 135°: $B\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$.

C is on the terminal arm of 225°: $C\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$.

D is on the terminal arm of 315°: $D\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$.

Section 2.2 Page 98 Question 21

a)

Angle	Sine	Cosine	Tangent
0°	0	1	0
15°	0.2588	0.9659	0.2679
30°	0.5	0.8660	0.5774
45°	0.7071	0.7071	1
60°	0.8660	0.5	1.7321
75°	0.9659	0.2588	3.7321
90°	1	0	undefined
105°	0.9659	−0.2588	−3.7321
120°	0.8660	−0.5	−1.7321
135°	0.7071	−0.7071	−1
150°	0.5	−0.8660	−0.5774
165°	0.2588	−0.9659	−0.2679
180°	0	−1	0

b) As θ increases from 0° to 90° the sine ratio increases from 0 to 1, while the cosine ratio decreases from 1 to 0. As θ increases from 90° to 180° the sine ratio decreases from 1 to 0, while the cosine ratio decreases from 0 to −1. As θ increases from 0° to 90° the tangent ratio increases from 0 to undefined, then from 90° to 180° the tangent ratio starts with large negative values but increases to 0.

c) The values of sine and cosine seem to be related. For example, sin 30° = cos 60°. However in quadrant II, the sign is different. For example, −sin 120° = cos 150°. For $0° \le \theta \le 90°$, $\cos\theta = \sin(90° - \theta)$. For $90° \le \theta \le 180°$, $\cos\theta = -\sin(\theta - 90°)$.

d) In quadrant I, sine, cosine, and tangent values are all positive. In quadrant II, sine values are positive but cosine and tangent values are negative.

e) In quadrant III, the sine and cosine values will be negative, but the tangent values will be positive. In quadrant IV, the cosine values will be positive, but the sine and tangent values will be negative.