Section 4.2 Page 230 Question 6

a) Let $r = 5b - 3$.

$4(5b-3)^2 + 10(5b-3) - 6$

$= 4r^2 + 10r - 6$

$= 2(2r^2 + 5r - 3)$

$= 2(2r-1)(r+3)$

$= 2(2(5b-3)-1)(5b-3+3)$

$= 2(10b-7)(5b)$

b) Use the pattern for factoring a difference of squares.

$16(x^2+1)^2 - 4(2x)^2$

$= [4(x^2+1) - 2(2x)][4(x^2+1) + 2(2x)]$

$= (4x^2 + 4 - 4x)(4x^2 + 4 + 4x)$

$= 16(x^2 - x + 1)(x^2 + x + 1)$

c) $-\frac{1}{4}(2x)^2 + 25(2y^3)^2$

$= -\frac{1}{4}\left[(2x)^2 - 100(2y^3)^2\right]$

$= -\frac{1}{4}[2x - 10(2y^3)][2x + 10(2y^3)]$

$= -\frac{1}{4}(2x - 20y^3)(2x + 20y^3)$

$= -(x - 10y^3)(x + 10y^3)$ or $(10y^3 - x)(10y^3 + x)$

Section 4.2 Page 230 Question 7

a) $(x+3)(x+4) = 0$

$x + 3 = 0$ or $x + 4 = 0$

$x = -3$ $x = -4$

The roots are –3 and –4.

b) $(x-2)\left(x+\frac{1}{2}\right) = 0$

$x - 2 = 0$ or $x + \frac{1}{2} = 0$

$x = 2$ $x = -\frac{1}{2}$

The roots are 2 and $-\frac{1}{2}$.

c) $(x + 7)(x - 8) = 0$

$x + 7 = 0$ or $x - 8 = 0$

$x = -7$ $x = 8$

The roots are –7 and 8.

d) $x(x + 5) = 0$

$x = 0$ or $x + 5 = 0$

$x = -5$

The roots are 0 and –5.

e) $(3x + 1)(5x - 4) = 0$

$3x + 1 = 0$ or $5x - 4 = 0$

$3x = -1$ $5x = 4$

$x = -\frac{1}{3}$ $x = \frac{4}{5}$

The roots are $-\frac{1}{3}$ and $\frac{4}{5}$.

f) $2(x - 4)(7 - 2x) = 0$

$x - 4 = 0$ or $7 - 2x = 0$

$x = 4$ $-2x = -7$

$x = \frac{7}{2}$

The roots are 4 and $\frac{7}{2}$.

Section 4.2 Page 230 Question 8

a) $10n^2 - 40 = 0$

$10(n^2 - 4) = 0$

$10(n - 2)(n + 2) = 0$

$n - 2 = 0$ or $n + 2 = 0$

$n = 2$ $n = -2$

For $n = 2$:

Left Side	Right Side
$10n^2 - 40$	0
$= 10(2)^2 - 40$	
$= 40 - 40$	
$= 0$	

Left Side = Right Side

For $n = -2$:

Left Side	Right Side
$10n^2 - 40$	0
$= 10(-2)^2 - 40$	
$= 40 - 40$	
$= 0$	

Left Side = Right Side

The roots are 2 and –2.

b) $\frac{1}{4}x^2 + \frac{5}{4}x + 1 = 0$

$\frac{1}{4}(x^2 + 5x + 4) = 0$

$\frac{1}{4}(x + 4)(x + 1) = 0$

$x + 4 = 0$ or $x + 1 = 0$

$x = -4$ $x = -1$

For $x = -4$:

Left Side	Right Side
$\frac{1}{4}x^2 + \frac{5}{4}x + 1$	0
$= \frac{1}{4}(-4)^2 + \frac{5}{4}(-4) + 1$	
$= 4 - 5 + 1$	
$= 0$	

Left Side = Right Side

For $x = -1$:

Left Side	Right Side
$\frac{1}{4}x^2 + \frac{5}{4}x + 1$	0
$= \frac{1}{4}(-1)^2 + \frac{5}{4}(-1) + 1$	
$= \frac{1}{4} - \frac{5}{4} + 1$	
$= 0$	

Left Side = Right Side

The roots are –4 and –1.

c) $3w^2 + 28w + 9 = 0$

$(3w + 1)(w + 9) = 0$

$3w + 1 = 0$ or $w + 9 = 0$

$3w = -1$ $w = -9$

$w = -\frac{1}{3}$

For $w = -\frac{1}{3}$:

Left Side	Right Side
$3w^2 + 28w + 9$	0
$= 3\left(-\frac{1}{3}\right)^2 + 28\left(-\frac{1}{3}\right) + 9$	
$= \frac{1}{3} - \frac{28}{3} + \frac{27}{3}$	
$= 0$	

Left Side = Right Side

For $w = -9$:

Left Side	Right Side
$3w^2 + 28w + 9$	0
$= 3(-9)^2 + 28(-9) + 9$	
$= 243 - 252 + 9$	
$= 0$	

Left Side = Right Side

The roots are $-\frac{1}{3}$ and –9.

d) $8y^2 - 22y + 15 = 0$

$(4y - 5)(2y - 3) = 0$

$4y - 5 = 0$ or $2y - 3 = 0$

$4y = 5$ $\quad$ $2y = 3$

$y = \frac{5}{4}$ $\quad$ $y = \frac{3}{2}$

For $y = \frac{5}{4}$:

Left Side	Right Side
$8y^2 - 22y + 15$	0
$= 8\left(\frac{5}{4}\right)^2 - 22\left(\frac{5}{4}\right) + 15$	
$= \frac{25}{2} - \frac{55}{2} + \frac{30}{2}$	
$= 0$	

Left Side = Right Side

For $y = \frac{3}{2}$:

Left Side	Right Side
$8y^2 - 22y + 15$	0
$= 8\left(\frac{3}{2}\right)^2 - 22\left(\frac{3}{2}\right) + 15$	
$= 18 - 33 + 15$	
$= 0$	

Left Side = Right Side

The roots are $\frac{5}{4}$ and $\frac{3}{2}$.

e) $d^2 + \frac{5}{2}d + \frac{3}{2} = 0$

$\frac{1}{2}(2d^2 + 5d + 3) = 0$

$\frac{1}{2}(2d + 3)(d + 1) = 0$

$2d + 3 = 0$ or $d + 1 = 0$

$2d = -3$ $\quad$ $d = -1$

$d = -\frac{3}{2}$

For $d = -\frac{3}{2}$:

Left Side	Right Side
$d^2 + \frac{5}{2}d + \frac{3}{2}$	0
$= \left(-\frac{3}{2}\right)^2 + \frac{5}{2}\left(-\frac{3}{2}\right) + \frac{3}{2}$	
$= \frac{9}{4} - \frac{15}{4} + \frac{6}{4}$	
$= 0$	

Left Side = Right Side

For $y = -1$:

Left Side	Right Side
$d^2 + \frac{5}{2}d + \frac{3}{2}$	0
$= (-1)^2 + \frac{5}{2}(-1) + \frac{3}{2}$	
$= \frac{2}{2} - \frac{5}{2} + \frac{3}{2}$	
$= 0$	

Left Side = Right Side

The roots are $-\frac{3}{2}$ and -1.

f) $4x^2 - 12x + 9 = 0$

$(2x - 3)(2x - 3) = 0$

$2x - 3 = 0$

$2x = 3$

$x = \frac{3}{2}$

For $x = \frac{3}{2}$:

Left Side	Right Side
$4x^2 - 12x + 9$	0

$= 4\left(\frac{3}{2}\right)^2 - 12\left(\frac{3}{2}\right) + 9$

$= 9 - 18 + 9$

$= 0$

Left Side = Right Side

The root is $\frac{3}{2}$.

Section 4.2 Page 230 Question 9

a) $k^2 - 5k = 0$

$k(k - 5) = 0$

$k = 0$ or $k - 5 = 0$

$k = 5$

For $k = 0$:

Left Side	Right Side
$k^2 - 5k$	0

$= 0^2 - 5(0)$

$= 0 - 0$

$= 0$

Left Side = Right Side

For $k = 5$:

Left Side	Right Side
$k^2 - 5k$	0

$= 5^2 - 5(5)$

$= 25 - 25$

$= 0$

Left Side = Right Side

The roots are 0 and 5.

b) $9x^2 = x + 8$

$9x^2 - x - 8 = 0$

$(9x + 8)(x - 1) = 0$

$9x + 8 = 0$ or $x - 1 = 0$

$9x = -8$ $x = 1$

$x = -\frac{8}{9}$

For $x = -\frac{8}{9}$:

Left Side	Right Side
$9x^2 - x - 8$	0
$= 9\left(-\frac{8}{9}\right)^2 - \left(-\frac{8}{9}\right) - 8$	
$= \frac{64}{9} + \frac{8}{9} - \frac{72}{9}$	
$= 0$	
Left Side = Right Side	

For $x = 1$:

Left Side	Right Side
$9x^2 - x - 8$	0
$= 9(1)^2 - 1 - 8$	
$= 9 - 9$	
$= 0$	
Left Side = Right Side	

The roots are $-\frac{8}{9}$ and 1.

c) $\frac{8}{3}t + 5 = -\frac{1}{3}t^2$

$\frac{1}{3}t^2 + \frac{8}{3}t + 5 = 0$

$\frac{1}{3}(t^2 + 8t + 15) = 0$

$\frac{1}{3}(t + 3)(t + 5) = 0$

$t + 3 = 0$ or $t + 5 = 0$

$t = -3$ $t = -5$

For $t = -3$:

Left Side	Right Side
$\frac{8}{3}t + 5$	$-\frac{1}{3}t^2$
$= \frac{8}{3}(-3) + 5$	$= -\frac{1}{3}(-3)^2$
$= -8 + 5$	$= -3$
$= -3$	
Left Side = Right Side	

For $t = -5$:

Left Side	Right Side
$\frac{8}{3}t + 5$	$-\frac{1}{3}t^2$
$= \frac{8}{3}(-5) + 5$	$= -\frac{1}{3}(-5)^2$
$= -\frac{40}{3} + \frac{15}{3}$	$= -\frac{25}{3}$
$= -\frac{25}{3}$	
Left Side = Right Side	

The roots are –3 and –5.

d) $\frac{25}{49}y^2 - 9 = 0$

$\left(\frac{5}{7}y - 3\right)\left(\frac{5}{7}y + 3\right) = 0$

$\frac{5}{7}y - 3 = 0$ or $\frac{5}{7}y + 3 = 0$

$\frac{5}{7}y = 3$ $\frac{5}{7}y = -3$

$y = \frac{21}{5}$ $y = -\frac{21}{5}$

For $y = \frac{21}{5}$:

Left Side	Right Side
$\frac{25}{49}y^2 - 9$	0
$= \frac{25}{49}\left(\frac{21}{5}\right)^2 - 9$	
$= \frac{441}{49} - \frac{441}{49}$	
$= 0$	

Left Side = Right Side

For $y = -\frac{21}{5}$:

Left Side	Right Side
$\frac{25}{49}y^2 - 9$	0
$= \frac{25}{49}\left(-\frac{21}{5}\right)^2 - 9$	
$= \frac{441}{49} - \frac{441}{49}$	
$= 0$	

Left Side = Right Side

The roots are $\frac{21}{5}$ and $-\frac{21}{5}$.

e) $2s^2 - 4s = 70$

$2s^2 - 4s - 70 = 0$

$2(s^2 - 2s - 35) = 0$

$2(s - 7)(s + 5) = 0$

$s - 7 = 0$ or $s + 5 = 0$

$s = 7$ $s = -5$

For $s = 7$:

Left Side	Right Side
$2s^2 - 4s$	70
$= 2(7)^2 - 4(7)$	
$= 98 - 28$	
$= 70$	

Left Side = Right Side

For $s = -5$:

Left Side	Right Side
$2s^2 - 4s$	70
$= 2(-5)^2 - 4(-5)$	
$= 50 + 20$	
$= 70$	

Left Side = Right Side

The roots are 7 and –5.

f) $4q^2 - 28q = -49$

$4q^2 - 28q + 49 = 0$

$(2q - 7)(2q - 7) = 0$

$2q - 7 = 0$

$2q = 7$

$q = \frac{7}{2}$

For $q = \frac{7}{2}$:

Left Side	Right Side
$4q^2 - 28q$	-49

$= 4\left(\frac{7}{2}\right)^2 - 28\left(\frac{7}{2}\right)$

$= 49 - 98$

$= -49$

Left Side = Right Side

The root is $\frac{7}{2}$.

Section 4.2 Page 230 Question 10

a) $42 = x^2 - x$

$0 = x^2 - x - 42$

$0 = (x - 7)(x + 6)$

$x - 7 = 0$ or $x + 6 = 0$

$x = 7$ $x = -6$

The roots are 7 and –6.

b) $g^2 = 30 - 7g$

$g^2 + 7g - 30 = 0$

$(g - 3)(g + 10) = 0$

$g - 3 = 0$ or $g + 10 = 0$

$g = 3$ $g = -10$

The roots are 3 and –10.

c) $y^2 + 4y = 21$

$y^2 + 4y - 21 = 0$

$(y - 3)(y + 7) = 0$

$y - 3 = 0$ or $y + 7 = 0$

$y = 3$ $y = -7$

The roots are 3 and –7.

d) $3 = 6p^2 - 7p$

$6p^2 - 7p - 3 = 0$

$(3p + 1)(2p - 3) = 0$

$3p + 1 = 0$ or $2p - 3 = 0$

$3p = -1$ $\quad$ $2p = 3$

$p = -\frac{1}{3}$ $\quad$ $p = \frac{3}{2}$

The roots are $-\frac{1}{3}$ and $\frac{3}{2}$.

e) $3x^2 + 9x = 30$

$3x^2 + 9x - 30 = 0$

$3(x^2 + 3x - 10) = 0$

$3(x - 2)(x + 5) = 0$

$x - 2 = 0$ or $x + 5 = 0$

$x = 2$ $\quad$ $x = -5$

The roots are 2 and –5.

f) $2z^2 = 3 - 5z$

$2z^2 + 5z - 3 = 0$

$(2z - 1)(z + 3) = 0$

$2z - 1 = 0$ or $z + 3 = 0$

$2z = 1$ $\quad$ $z = -3$

$z = \frac{1}{2}$

The roots are $\frac{1}{2}$ and –3.

Section 4.2 Page 230 Question 11

a) Substitute the dimensions and given area into $A = \ell w$:

$54 = (x + 10)(2x - 3)$

$54 = 2x^2 + 17x - 30$

$0 = 2x^2 + 17x - 84$

b) Solve the equation from part a) to find the value of x.

$0 = 2x^2 + 17x - 84$

$0 = (2x - 7)(x + 12)$

$2x - 7 = 0$ or $x + 12 = 0$

$2x = 7$ $\quad$ $x = -12$

$x = \frac{7}{2}$

Since x represents a distance, it cannot be negative. So, reject the root –12.

The value of x is $\frac{7}{2}$, or 3.5 cm.