f) Complete the square to determine the maximum or minimum value.

$f(x) = –2x^2 + 5.8x – 3$
$f(x) = –2(x^2 – 2.9x) – 3$
$f(x) = –2(x^2 – 2.9x + 2.1025 – 2.1025) – 3$
$f(x) = –2[(x^2 – 2.9x + 2.1025) – 2.1025] – 3$
$f(x) = –2[(x – 1.45)^2 – 2.1025] – 3$
$f(x) = –2(x – 1.45)^2 + 4.205 – 3$
$f(x) = –2(x – 1.45)^2 + 1.205$

Since $a < 0$, the graph has a maximum value of 1.205.

Section 3.3 Page 193 Question 8

a) Convert $y = x^2 + \frac{3}{2}x – 7$ to vertex form.

$$y = x^2 + \frac{3}{2}x - 7$$
$$y = \left(x^2 + \frac{3}{2}x\right) - 7$$
$$y = \left(x^2 + \frac{3}{2}x + \frac{9}{16} - \frac{9}{16}\right) - 7$$
$$y = \left(x^2 + \frac{3}{2}x + \frac{9}{16}\right) - \frac{9}{16} - 7$$
$$y = \left(x + \frac{3}{4}\right)^2 - \frac{121}{16}$$

b) Convert $y = –x^2 – \frac{3}{8}x$ to vertex form.

$$y = -x^2 + \frac{3}{8}x$$
$$y = -\left(x^2 - \frac{3}{8}x\right)$$
$$y = -\left(x^2 - \frac{3}{8}x + \frac{9}{256} - \frac{9}{256}\right)$$
$$y = -\left[\left(x^2 + \frac{3}{8}x + \frac{9}{256}\right) - \frac{9}{256}\right]$$
$$y = -\left[\left(x + \frac{3}{16}\right)^2 - \frac{9}{256}\right]$$
$$y = -\left(x + \frac{3}{16}\right)^2 + \frac{9}{256}$$

c) Convert $y = 2x^2 - \frac{5}{6}x + 1$ to vertex form.

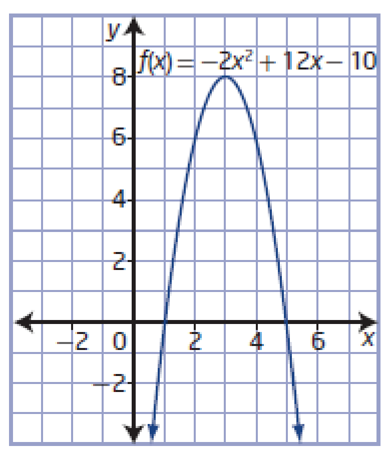
$$y = 2x^2 - \frac{5}{6}x + 1$$
$$y = 2\left(x^2 - \frac{5}{12}x\right) + 1$$
$$y = 2\left(x^2 - \frac{5}{12}x + \frac{25}{576} - \frac{25}{576}\right) + 1$$
$$y = 2\left[\left(x^2 - \frac{5}{12}x + \frac{25}{576}\right) - \frac{25}{576}\right] + 1$$
$$y = 2\left[\left(x - \frac{5}{24}\right)^2 - \frac{25}{576}\right] + 1$$
$$y = 2\left(x - \frac{5}{24}\right)^2 - \frac{25}{288} + 1$$
$$y = 2\left(x - \frac{5}{24}\right)^2 + \frac{263}{288}$$

Section 3.3 Page 193 Question 9

a) Convert $f(x) = -2x^2 + 12x - 10$ to vertex form.

$f(x) = -2x^2 + 12x - 10$
$f(x) = -2(x^2 - 6x) - 10$
$f(x) = -2(x^2 - 6x + 9 - 9) - 10$
$f(x) = -2[(x^2 - 6x + 9) - 9] - 10$
$f(x) = -2[(x - 3)^2 - 9] - 10$
$f(x) = -2(x - 3)^2 + 18 - 10$
$f(x) = -2(x - 3)^2 + 8$

b) Answers may vary. Example: The graph shows the vertex at (3, 8), which agrees with vertex form found in part a).



error in line 5. –3 was not distributed properly. In line 2, the coefficient of the x-term should be 2, then add and subtract 1.
$f(x) = -3x^2 - 6x$
$f(x) = -3(x^2 + 2x + 1 - 1)$
$f(x) = -3[(x^2 + 2x + 1) - 1]$
$f(x) = -3[(x + 1)^2 - 1]$
$f(x) = -3(x + 1)^2 + 3$

Section 3.3 Page 194 Question 13

Complete the square to determine the vertex of $C(n) = 75n^2 - 1800n + 60\ 000$.
$C(n) = 75n^2 - 1800n + 60\ 000$
$C(n) = -75(n^2 + 24n) + 60\ 000$
$C(n) = -75(n^2 + 24n + 144 - 144) + 60\ 000$
$C(n) = -75[(n^2 + 24n + 144) - 144] + 60\ 000$
$C(n) = -75[(n + 12)^2 - 144] + 60\ 000$
$C(n) = -75(n + 12)^2 + 10\ 800 + 60\ 000$
$C(n) = -75(n + 12)^2 + 70\ 800$
The business should produce 12 000 items to minimize their costs at $70 800.

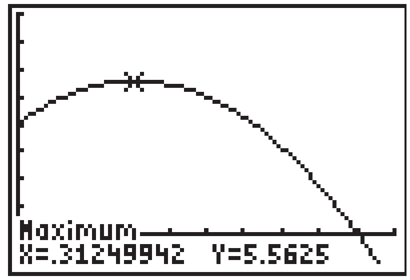
Section 3.3 Page 194 Question 14

Complete the square to determine the vertex of $h(t) = -5t^2 + 10t + 4$.
$h(t) = -5t^2 + 10t + 4$
$h(t) = -5(t^2 - 2t) + 4$
$h(t) = -5(t^2 - 2t + 1 - 1) + 4$
$h(t) = -5[(t^2 - 2t + 1) - 1] + 4$
$h(t) = -5[(t - 1)^2 - 1] + 4$
$h(t) = -5(t - 1)^2 + 5 + 4$
$h(t) = -5(t - 1)^2 + 9$
The maximum height of the gymnast on each jump is 9 m.

Section 3.3 Page 194 Question 15

a) Complete the square to determine the vertex of $h(t) = -16t^2 + 10t + 4$.
$h(t) = -16t^2 + 10t + 4$
$h(t) = -16(t^2 - 0.625t) + 4$
$h(t) = -16(t^2 - 0.625t + 0.3125^2 - 0.3125^2) + 4$
$h(t) = -16[(t^2 - 0.625t + 0.3125^2) - 0.3125^2] + 4$
$h(t) = -16[(t - 0.3125)^2 - 0.3125^2] + 4$
$h(t) = -16(t - 0.3125)^2 + 1.5625 + 4$
$h(t) = -16(t - 0.3125)^2 + 5.5625$
The maximum height of the arrow is 5.5625 ft 0.3125 s after it was fired.

b) Answers may vary. Examples:
Verify by graphing $h(t) = -16t^2 + 10t + 4$ and finding the vertex.



Alternatively, use $t = \dfrac{-b}{2a}$ to find the t-coordinate of the vertex.

$t = \dfrac{-10}{2(-16)}$

$t = 0.3125$

Substitute $t = 0.3125$ into $h(t) = -16t^2 + 10t + 4$ to find the h-coordinate of the vertex.

$h(0.3125) = -16(0.3125)^2 + 10(0.3125) + 4$

$h(0.3125) = 5.5625$

The vertex is located at (0.3125, 5.5625).

Section 3.3 Page 194 Question 16

a) *Austin's solution:*

$y = -6x^2 + 72x - 20$
$y = -6(x^2 + 12x) - 20$
$y = -6(x^2 + 12x + 36 - 36) - 20$
$y = -6[(x^2 + 12x + 36) - 36] - 20$
$y = -6[(x + 6) - 36] - 20$
$y = -6(x + 6) + 216 - 20$
$y = -6(x + 6) + 196$

There is an error in line 2. –6 was not correctly factored out of the coefficient of the x-term. The coefficient of the x-term should be –12. In line 5, the expression $(x + 6)$ was not squared.

$y = -6x^2 + 72x - 20$
$y = -6(x^2 - 12x) - 20$
$y = -6(x^2 - 12x + 36 - 36) - 20$
$y = -6[(x^2 - 12x + 36) - 36] - 20$
$y = -6[(x - 6)^2 - 36] - 20$
$y = -6(x - 6)^2 + 216 - 20$
$y = -6(x - 6)^2 + 196$

Yuri's solution:

$y = -6x^2 + 72x - 20$
$y = -6(x^2 - 12x) - 20$
$y = -6(x^2 - 12x + 36 - 36) - 20$
$y = -6[(x^2 - 12x + 36) - 36] - 20$

$y = –6[(x – 6)^2 – 36] – 20$
$y = –6(x – 6)^2 – 216 – 20$
$y = –6(x – 6)^2 + 236$
There is an error in line 6. –6 was not correctly distributed: –6(–36) = 216. Then, the corrected function is $y = –6(x – 6)^2 + 196$.

b) Answers may vary. Example: To verify an answer, either work backward to show the functions are equivalent or use technology to show the graphs of the functions are identical. In either case, Austin and Yuri would have found that the functions and graphs were not equivalent.

Section 3.3 Page 195 Question 17

Complete the square to determine the vertex of $d(x) = 0.03125x^2 – 1.5x$.
$d(x) = 0.03125x^2 – 1.5x$
$d(x) = 0.03125(x^2 – 48x)$
$d(x) = 0.03125(x^2 – 48x + 576 – 576)$
$d(x) = 0.03125[(x^2 – 48x + 576) – 576]$
$d(x) = 0.03125[(x – 24)^2 – 576]$
$d(x) = 0.03125(x – 24)^2 – 18$
The dish has a depth of 18 cm at its centre.

Section 3.3 Page 195 Question 18

a) Write a function to model this situation.
Let n represent the number of price decreases. The new price is \$70 minus the number of price decreases times \$1, or $70 – n$.
The new number of tickets sold is 2000 plus the number of price decreases times 50, or $2000 + 50n$.
Let R represent the expected revenue, in dollars.
Revenue = (price)(number of sessions)
$R = (70 – n)(2000 + 50n)$
$R = 140\ 000 + 1500n – 50n^2$
$R = –50n^2 + 1500n + 140\ 000$
Complete the square to find the vertex.
$R = –50n^2 + 1500n + 140\ 000$
$R = –50(n^2 – 30n) + 140\ 000$
$R = –50(n^2 – 30n + 225 – 225) + 140\ 000$
$R = –50[(n^2 – 30n + 225) – 225] + 140\ 000$
$R = –50[(n – 15)^2 – 225] + 140\ 000$
$R = –50(n – 15)^2 + 11\ 250 + 140\ 000$
$R = –50(n – 15)^2 + 151\ 250$
The maximum revenue the promoter can expect is \$151 250 when the ticket price is 70 – 15, or \$55.