b) Complete the square to write $y = x^2 - 18x - 59$ in vertex form.
$y = x^2 - 18x - 59$
$y = (x^2 - 18x) - 59$
$y = (x^2 - 18x + 81 - 81) - 59$
$y = (x^2 - 18x + 81) - 81 - 59$
$y = (x - 9)^2 - 81 - 59$
$y = (x - 9)^2 - 140$
The vertex of the function is (9, –140).

c) Complete the square to write $y = x^2 - 10x + 31$ in vertex form.
$y = x^2 - 10x + 31$
$y = (x^2 - 10x) + 31$
$y = (x^2 - 10x + 25 - 25) + 31$
$y = (x^2 - 10x + 25) - 25 + 31$
$y = (x - 5)^2 - 25 + 31$
$y = (x - 5)^2 + 6$
The vertex of the function is (5, 6).

d) Complete the square to write $y = x^2 + 32x - 120$ in vertex form.
$y = x^2 + 32x - 120$
$y = (x^2 + 32x) - 120$
$y = (x^2 + 32x + 256 - 256) - 120$
$y = (x^2 + 32x + 256) - 256 - 120$
$y = (x + 16)^2 - 256 - 120$
$y = (x + 16)^2 - 376$
The vertex of the function is (–16, –376).

Section 3.3 Page 193 Question 3

a) Complete the square to write $y = 2x^2 - 12x$ in the form $y = a(x - p)^2 + q$.
$y = 2x^2 - 12x$
$y = 2(x^2 - 6x)$
$y = 2(x^2 - 6x + 9 - 9)$
$y = 2[(x^2 - 6x + 9) - 9]$
$y = 2[(x - 3)^2 - 9]$
$y = 2(x - 3)^2 - 18$
Expand $y = 2(x - 3)^2 - 18$ to verify the two forms are equivalent.
$y = 2(x - 3)^2 - 18$
$y = 2(x^2 - 6x + 9) - 18$
$y = 2x^2 - 12x + 18 - 18$
$y = 2x^2 - 12x$

b) Complete the square to write $y = 6x^2 + 24x + 17$ in the form $y = a(x - p)^2 + q$.

$y = 6x^2 + 24x + 17$

$y = 6(x^2 + 4x) + 17$

$y = 6(x^2 + 4x + 4 - 4) + 17$

$y = 6[(x^2 + 4x + 4) - 4] + 17$

$y = 6[(x + 2)^2 - 4] + 17$

$y = 6(x + 2)^2 - 24 + 17$

$y = 6(x + 2)^2 - 7$

Expand $y = 6(x + 2)^2 - 7$ to verify the two forms are equivalent.

$y = 6(x + 2)^2 - 7$

$y = 6(x^2 + 4x + 4) - 7$

$y = 6x^2 + 24x + 24 - 7$

$y = 6x^2 + 24x + 17$

c) Complete the square to write $y = 10x^2 - 160x + 80$ in the form $y = a(x - p)^2 + q$.

$y = 10x^2 - 160x + 80$

$y = 10(x^2 - 16x) + 80$

$y = 10(x^2 - 16x + 64 - 64) + 80$

$y = 10[(x^2 - 16x + 64) - 64] + 80$

$y = 10[(x - 8)^2 - 64] + 80$

$y = 10(x - 8)^2 - 640 + 80$

$y = 10(x - 8)^2 - 560$

Expand $y = 10(x - 8)^2 - 560$ to verify the two forms are equivalent.

$y = 10(x - 8)^2 - 560$

$y = 10(x^2 - 16x + 64) - 560$

$y = 10x^2 - 160x + 640 - 560$

$y = 10x^2 - 160x + 80$

d) Complete the square to write $y = 3x^2 + 42x - 96$ in the form $y = a(x - p)^2 + q$.

$y = 3x^2 + 42x - 96$

$y = 3(x^2 + 14x) - 96$

$y = 3(x^2 + 14x + 49 - 49) - 96$

$y = 3[(x^2 + 14x + 49) - 49] - 96$

$y = 3[(x + 7)^2 - 49] - 96$

$y = 3(x + 7)^2 - 147 - 96$

$y = 3(x + 7)^2 - 243$

Expand $y = 3(x + 7)^2 - 243$ to verify the two forms are equivalent.

$y = 3(x + 7)^2 - 243$

$y = 3(x^2 + 14x + 49) - 243$

$y = 3x^2 + 42x + 147 - 243$

$y = 3x^2 + 42x - 96$

Section 3.3 Page 193 Question 4

a) Covert $f(x) = -4x^2 + 16x$ to vertex form.

$f(x) = -4x^2 + 16x$
$f(x) = -4(x^2 - 4x)$
$f(x) = -4(x^2 - 4x + 4 - 4)$
$f(x) = -4[(x^2 - 4x + 4) - 4]$
$f(x) = -4[(x - 2)^2 - 4]$
$f(x) = -4(x - 2)^2 + 16$

Expand $f(x) = -4(x - 2)^2 + 16$ to verify the two forms are equivalent.

$f(x) = -4(x - 2)^2 + 16$
$f(x) = -4(x^2 - 4x + 4) + 16$
$f(x) = -4x^2 + 16x - 16 + 16$
$f(x) = -4x^2 + 16x$

b) Covert $f(x) = -20x^2 - 400x - 243$ to vertex form.

$f(x) = -20x^2 - 400x - 243$
$f(x) = -20(x^2 + 20x) - 243$
$f(x) = -20(x^2 + 20x + 100 - 100) - 243$
$f(x) = -20[(x^2 + 20x + 100) - 100] - 243$
$f(x) = -20[(x + 10)^2 - 100] - 243$
$f(x) = -20(x + 10)^2 + 2000 - 243$
$f(x) = -20(x + 10)^2 + 1757$

Expand $f(x) = -20(x + 10)^2 + 1757$ to verify the two forms are equivalent.

$f(x) = -20(x + 10)^2 + 1757$
$f(x) = -20(x^2 + 20x + 100) + 1757$
$f(x) = -20x^2 - 400x - 2000 + 1757$
$f(x) = -20x^2 - 400x - 243$

c) Covert $f(x) = -x^2 - 42x + 500$ to vertex form.

$f(x) = -x^2 - 42x + 500$
$f(x) = -(x^2 + 42x) + 500$
$f(x) = -(x^2 + 42x + 441 - 441) + 500$
$f(x) = -[(x^2 + 42x + 441) - 441] + 500$
$f(x) = -[(x + 21)^2 - 441] + 500$
$f(x) = -(x + 21)^2 + 441 + 500$
$f(x) = -(x + 21)^2 + 941$

Expand $f(x) = -(x + 21)^2 + 941$ to verify the two forms are equivalent.

$f(x) = -(x + 21)^2 + 941$
$f(x) = -(x^2 + 42x + 441) + 941$
$f(x) = -x^2 - 42x + 500$

d) Covert $f(x) = -7x^2 + 182x - 70$ to vertex form.

$f(x) = -7x^2 + 182x - 70$

$f(x) = -7(x^2 - 26x) - 70$

$f(x) = -7(x^2 - 26x + 169 - 169) - 70$

$f(x) = -7[(x^2 - 26x + 169) - 169] - 70$

$f(x) = -7[(x - 13)^2 - 169] - 70$

$f(x) = -7(x - 13)^2 + 1183 - 70$

$f(x) = -7(x - 13)^2 + 1113$

Expand $f(x) = -7(x - 13)^2 + 1113$ to verify the two forms are equivalent.

$f(x) = -7(x - 13)^2 + 1113$

$f(x) = -7(x^2 - 26x + 169) + 1113$

$f(x) = -7x^2 + 182x - 1183 + 1113$

$f(x) = -7x^2 + 182x - 70$

Section 3.3 Page 193 Question 5

a) Verify that $y = x^2 - 22x + 13$ and $y = (x - 11)^2 - 108$ represent the same function.

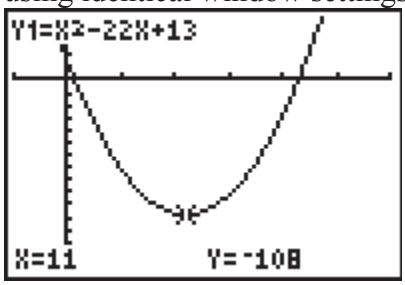
Algebraically: Expand $y = (x - 11)^2 - 108$ and compare to $y = x^2 - 22x + 13$.

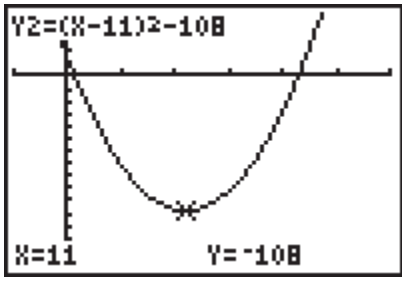
$y = (x - 11)^2 - 108$

$y = x^2 - 22x + 121 - 108$

$y = x^2 - 22x + 13$

Graphically: Use a graphing calculator to graph both functions together or separately using identical window settings.





b) Verify that $y = 4x^2 + 120x$ and $y = 4(x + 15)^2 - 900$ represent the same function, algebraically and graphically.

Algebraically:

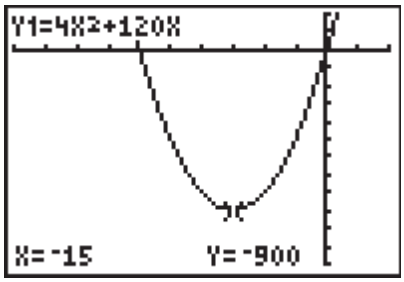
$y = 4(x + 15)^2 - 900$

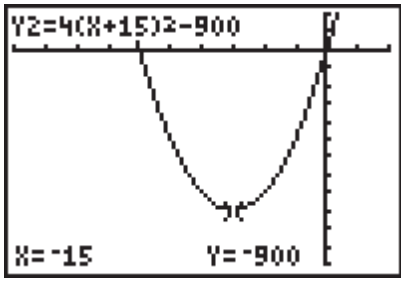
$y = 4(x^2 + 30x + 225) - 900$

$y = 4x^2 + 120x + 900 - 900$

$y = 4x^2 + 120x$

Graphically:





c) Verify that $y = 9x^2 - 54x - 10$ and $y = 9(x - 3)^2 - 91$ represent the same function, algebraically and graphically.

Algebraically:

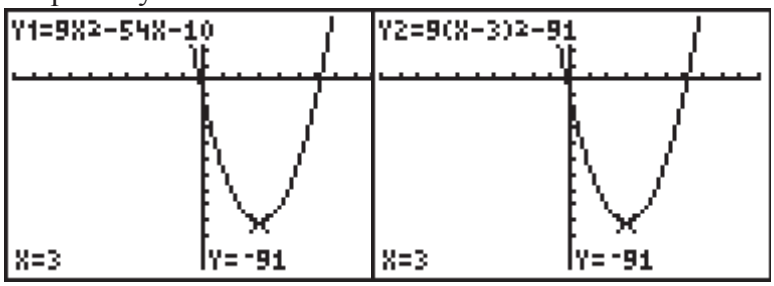
$y = 9(x - 3)^2 - 91$

$y = 9(x^2 - 6x + 9) - 91$

$y = 9x^2 - 54x + 81 - 91$

$y = 9x^2 - 54x - 10$

Graphically:



d) Verify that $y = -4x^2 - 8x + 2$ and $y = -4(x + 1)^2 + 6$ represent the same function, algebraically and graphically.

Algebraically:

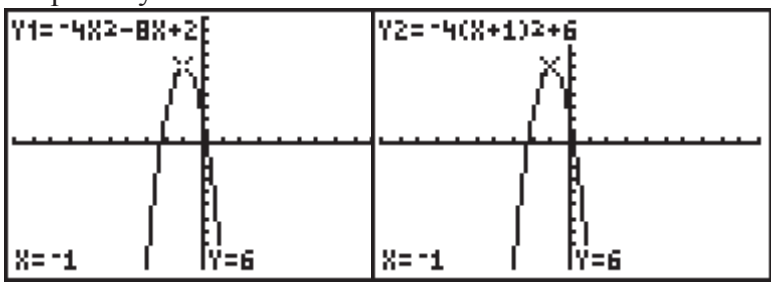
$y = -4(x + 1)^2 + 6$

$y = -4(x^2 + 2x + 1) + 6$

$y = -4x^2 - 8x - 4 + 6$

$y = -4x^2 - 8x + 2$

Graphically:



Section 3.3 Page 193 Question 6

a) Complete the square to determine the maximum or minimum value of $y = x^2 + 6x - 2$.

$y = x^2 + 6x - 2$

$y = (x^2 + 6x + 9 - 9) - 2$

$y = (x^2 + 6x + 9) - 9 - 2$

$y = (x + 3)^2 - 11$

Since $a > 0$, the graph has a minimum value of -11 when $x = -3$.