Section 3.1 Page 158 Question 9

a) vertex at (0, 0), passing through the point (6, −9)
Since $p = 0$ and $q = 0$, the function is of the form $y = ax^2$.
Substitute the coordinates of the given point to find a.
$-9 = a(6)^2$
$-9 = 36a$
$a = -\frac{1}{4}$

The quadratic function in vertex form with the given characteristics is $y = -\frac{1}{4}x^2$.

b) vertex at (0, –6), passing through the point (3, 21)
Since $p = 0$ and $q = –6$, the function is of the form $y = ax^2 – 6$.
Substitute the coordinates of the given point to find a.
$21 = a(3)^2 – 6$
$27 = 9a$
$a = 3$
The quadratic function in vertex form with the given characteristics is $y = 3x^2 – 6$.

c) vertex at (2, 5), passing through the point (4, –11)
Since $p = 2$ and $q = 5$, the function is of the form $y = a(x – 2)^2 + 5$.
Substitute the coordinates of the given point to find a.
$–11 = a(4 – 2)^2 + 5$
$–16 = 4a$
$a = –4$
The quadratic function in vertex form with the given characteristics is $y = –4(x – 2)^2 + 5$.

d) vertex at (–3, –10), passing through the point (2, –5)
Since $p = –3$ and $q = –10$, the function is of the form $y = a(x + 3)^2 – 10$.
Substitute the coordinates of the given point to find a.
$–5 = a(2 + 3)^2 – 10$
$5 = 25a$
$a = \frac{1}{5}$

The quadratic function in vertex form with the given characteristics is $y = \frac{1}{5}(x + 3)^2 – 10$.

Section 3.1 Page 158 Question 10

a) A horizontal translation of 5 units to the left causes the x-coordinate of the given point (4, 16) to change by –5.
$(4, 16) \rightarrow (4 - 5, 16)$ or $(-1, 16)$
Then, a vertical translation of 8 units up causes the y-coordinate of the point (–1, 16) to change by +8.
$(-1, 16) \rightarrow (-1, 16 + 8)$ or $(-1, 24)$

b) A change in width by a factor of $\frac{1}{4}$ causes the y-coordinate of the given point (4, 16) to change by a factor of $\frac{1}{4}$.
$(4, 16) \rightarrow \left(4, \frac{1}{4} \times 16\right)$ or $(4, 4)$
Then, a reflection in the x-axis causes the y-coordinate of the point (4, 4) to change by a factor of –1.
$(4, 4) \rightarrow (4, -1 \times 4)$ or $(4, -4)$

c) A reflection in the x-axis causes the y-coordinate of the given point (4, 16) to change by a factor of –1.
$(4, 16) \rightarrow (4, -1 \times 16)$ or $(4, -16)$
Then, a horizontal translation of 10 units to the right causes the x-coordinate of the point (4, –16) to change by +10.
$(4, -16) \rightarrow (4 + 10, -16)$ or $(14, -16)$

d) A change in width by a factor of 3 causes the y-coordinate of the given point (4, 16) to change by a factor of 3.
$(4, 16) \rightarrow (4, 3 \times 16)$ or $(4, 48)$
Then, a vertical translation of 8 units down causes the y-coordinate of the point (4, 48) to change by –8.
$(4, 48) \rightarrow (4, 48 - 8)$ or $(4, 40)$

Section 3.1 Page 158 Question 11

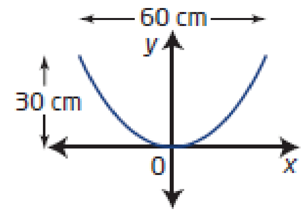
First rewrite the quadratic function $y = 20 - 5x^2$ in vertex form: $y = -5x^2 + 20$.
So, $a = -5$, $p = 0$, and $q = 20$.
To obtain the graph of $y = -5x^2 + 20$, transform the graph of $y = x^2$ by
- multiplying the y-values by a factor of 5
- reflecting in the x-axis
- translating 20 units up

Section 3.1 Page 158 Question 12

No. Quadratic functions will always have one y-intercept. Since the graphs have a domain of $\{x \mid x \in \mathrm{R}\}$, the parabola will always intersect the y-axis.

Section 3.1 Page 159 Question 13

a) Let x and y represent the horizontal and vertical distances from the low point at the centre of the mirror, respectively. Since the vertex is at (0, 0), the quadratic function is of the form $y = ax^2$. From the diagram, another point on the parabola is (30, 30).



Use the coordinates of this point to find a.

$30 = a(30)^2$

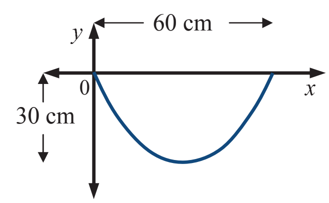
$30 = 900a$

$a = \dfrac{1}{30}$

A quadratic function that represents the cross-sectional shape is $y = \dfrac{1}{30}x^2$.

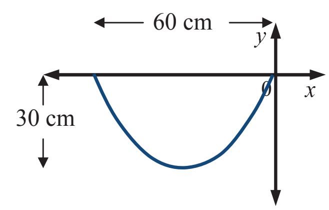
b) If the left outer edge of the mirror is considered the origin, then the vertex is at (30, –30) and the quadratic function becomes

$y = \dfrac{1}{30}(x - 30)^2 - 30$.



If the right outer edge of the mirror is considered the origin, then the vertex is at (–30, –30) and the quadratic function becomes

$y = \dfrac{1}{30}(x + 30)^2 - 30$.



Section 3.1 Page 159 Question 14

a) For $N(x) = -2.5(x - 36)^2 + 20\,000$, $a = -2.5$, $p = 36$, and $q = 20\,000$. To sketch the graph of $N(x) = -2.5(x - 36)^2 + 20\,000$, transform the graph of $y = x^2$ by

- multiplying the y-values by a factor of 2.5
- reflecting in the x-axis
- translating 36 units to the right and 20 000 units up

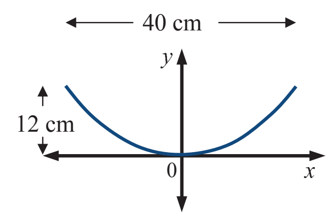
The graph of $N(x) = -2.5(x - 36)^2 + 20\,000$ has its vertex located at (36, 20 0000).
The equation of the axis of symmetry is $x = 36$.
Since $a < 0$, the graph opens downward and has a maximum value of 20 000.

b) The value of x that corresponds to the maximum value of the graph is 36. The optimum number of times that the commercial should be aired is 36 times.

c) The maximum value of the graph is the y-coordinate of the vertex, or 20 000. The maximum number of people that will buy this product is 20 000 people.

Section 3.1 Page 159 Question 15

Answers may vary. Example:



Choose the location of the origin to be the lowest point in the centre of the container. Let x and y represent the horizontal and vertical
distances from the low point of the container, respectively. Then, the vertex is at (0, 0) and the quadratic function is of the form $y = ax^2$. From the diagram, another point on the parabola is (20, 12).
Use the coordinates of this point to find a.

$12 = a(20)^2$

$12 = 400a$

$a = \frac{3}{100}$

A quadratic function that represents the cross-sectional shape is $y = \frac{3}{100}x^2$.

Section 3.1 Page 160 Question 16

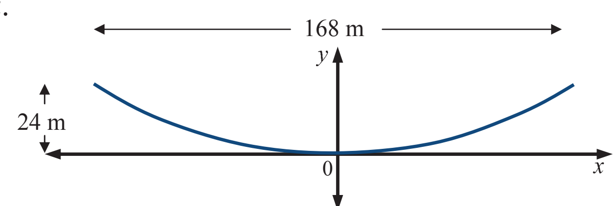
Answers may vary. Examples:

a) Choose the location of the origin to be the lowest point in the centre of the cables. Let x and y represent the horizontal and vertical distances from the low point of the cables, respectively. Then, the vertex is at (0, 0) and the quadratic function is of the form $y = ax^2$. From the diagram, another point on the parabola is (84, 24).
Use the coordinates of this point to find a.

$24 = a(84)^2$

$24 = 7056a$

$a = \frac{1}{294}$



A quadratic function that represents the shape of the cables is $y = \frac{1}{294}x^2$.

b) If the top of the left tower is considered the origin, then the vertex is at (84, –24).