Chapter 3

Quadratic Functions

Complete the following sentences.

- ☆ The graph of a quadratic function is a PARABOLA .
- ☆ The lowest point of the graph or the highest point of the graph is called a VERTEX .
- ☆ If the parabola opens upward, the *y*-coordinate of the vertex is called the MINIMUM POINT .
- ☆ If the parabola opens downward, the *y*-coordinate of the vertex is called the MAXIMUM POINT .
- ☆ The parabola is symmetric about a line called the AXIS OF SYMMETRY .

axis of symmetry maximum value minimum value

parabola vertex

Remember $y = ax^2 + bx + c$?

How did we find the

- direction of the opening?
- y-intercept ?
- x-intercepts ?
- equation of the axis of symmetry ?
- coordinates of the vertex ?
- domain and range ?

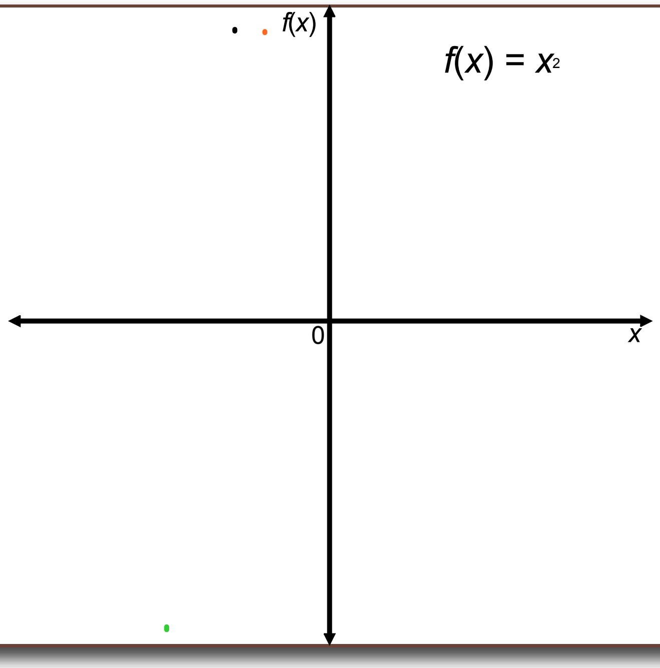
Chapter 3

The Effect of Parameter *a* in $f(x) = ax^2$ on the Graph of $f(x) = x^2$

Match the functions and the parabolas. Highlight the functions using the coloured boxes to indicate your matching.

$f(x) = -0.1x^2$

$f(x) = 2x^2$



$f(x) = x^2$

Check answer

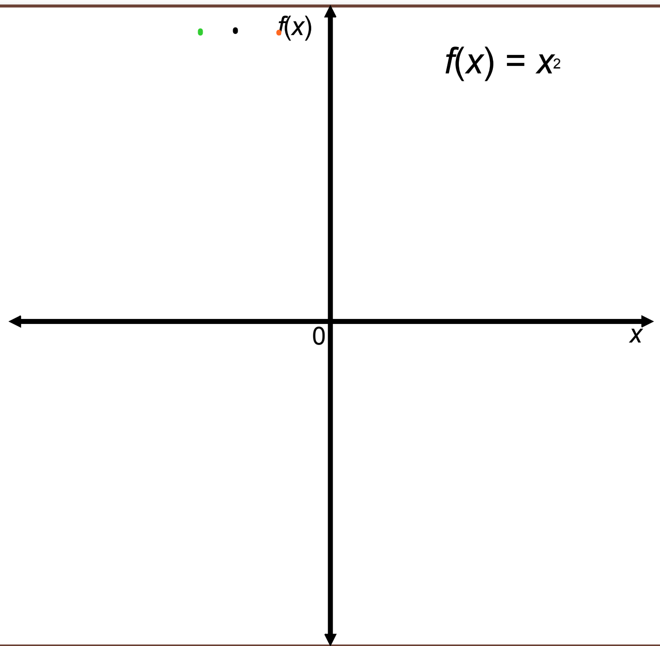
Chapter 3

The Effect of Parameter q in $f(x) = x^2 + q$ on the Graph of $f(x) = x^2$

Match the functions and the parabolas. Highlight the functions using the coloured boxes to indicate your matching.

$f(x) = x^2 + 3$

$f(x) = x^2 - 3$



$f(x) = x^2$

Check answer

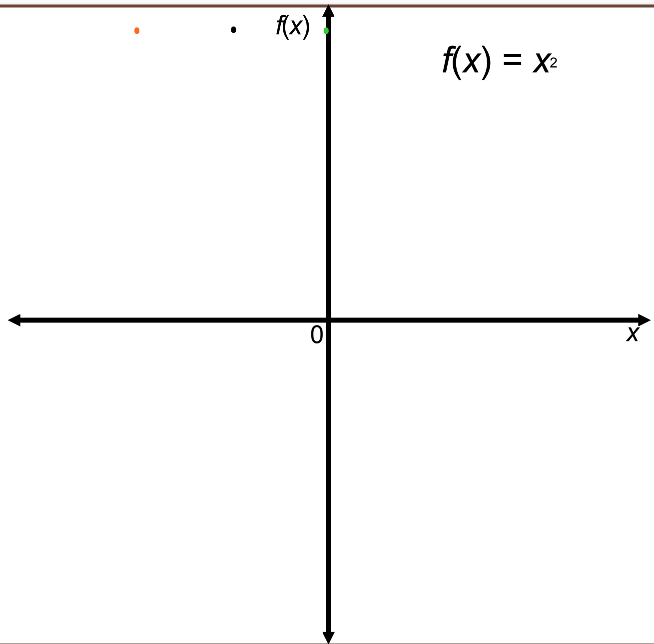
Chapter 3

The Effect of Parameter p in $f(x) = (x - p)^2$ on the Graph of $f(x) = x^2$

Match the functions and the parabolas. Highlight the functions using the coloured boxes to indicate your matching.

$f(x) = (x - 3)^2$

$f(x) = (x + 3)^2$

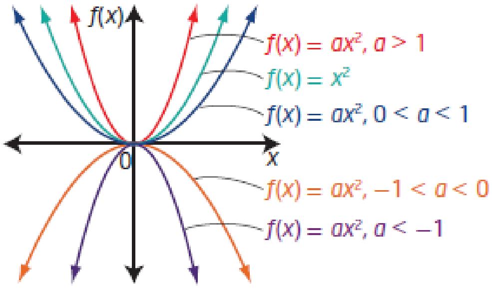


$f(x) = x^2$

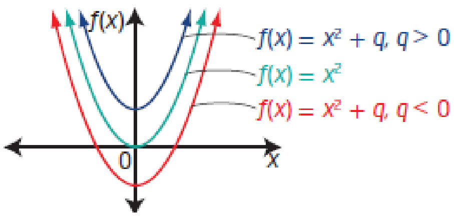
Check answer

Key Ideas

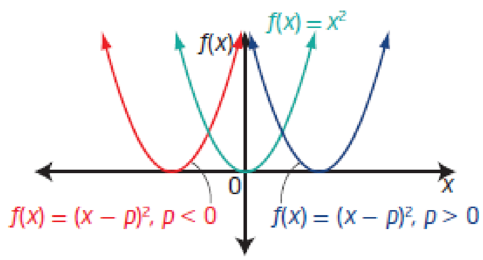
- For a quadratic function in vertex form, $f(x) = a(x - p)^2 + q$, $a \neq 0$, the graph:
  - has the shape of a parabola
  - has its vertex at (p, q)
  - has an axis of symmetry with equation $x = p$
  - is congruent to $f(x) = ax^2$ translated horizontally by p units and vertically by q units
- Sketch the graph of $f(x) = a(x - p)^2 + q$ by transforming the graph of $f(x) = x^2$.
  - The graph opens upward if $a > 0$.
  - If $a < 0$, the parabola is reflected in the x-axis; it opens downward.
  - If $-1 < a < 1$, the parabola is wider compared to the graph of $f(x) = x^2$.
  - If $a > 1$ or $a < -1$, the parabola is narrower compared to the graph of $f(x) = x^2$.



  - The parameter q determines the vertical position of the parabola.
  - If $q > 0$, then the graph is translated q units up.
  - If $q < 0$, then the graph is translated q units down.



  - The parameter p determines the horizontal position of the parabola.
  - If $p > 0$, then the graph is translated p units to the right.
  - If $p < 0$, then the graph is translated p units to the left.



Example 1

Sketch Graphs of Quadratic Functions in Vertex Form

Determine the following characteristics for each function.
- the vertex
- the domain and range
- the direction of opening
- the equation of the axis of symmetry

Then, sketch each graph.

a) $y = 2(x + 1)^2 - 3$ **b)** $y = -\frac{1}{4}(x - 4)^2 + 1$

Solution

a) Use the values of a, p, and q to determine some characteristics of $y = 2(x + 1)^2 - 3$ and sketch the graph.

$$y = 2(x + 1)^2 - 3$$

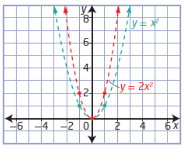
$a = 2$ $p = -1$ $q = -3$

Since $p = -1$ and $q = -3$, the vertex is located at $(-1, -3)$.
Since $a > 0$, the graph opens upward. Since $a > 1$, the parabola is narrower compared to the graph of $y = x^2$.
Since $q = -3$, the range is $\{y \mid y \geq -3, y \in R\}$.
The domain is $\{x \mid x \in R\}$.
Since $p = -1$, the equation of the axis of symmetry is $x = -1$.

Method 1: Sketch Using Transformations

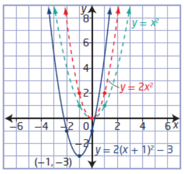
Sketch the graph of $y = 2(x + 1)^2 - 3$ by transforming the graph of $y = x^2$.
- Use the points (0, 0), (1, 1), and (−1, 1) to sketch the graph of $y = x^2$.
- Apply the change in width.

When using transformations to sketch the graph, you should deal with parameter *a* first, since its reference for wider or narrower is relative to the *y*-axis.



- Translate the graph.

How are *p* and *q* related to the direction of the translations and the location of the vertex?

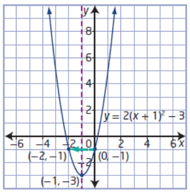


Method 2: Sketch Using Points and Symmetry
- Plot the coordinates of the vertex, (−1, −3), and draw the axis of symmetry, $x = -1$.
- Determine the coordinates of one other point on the parabola.

The y-intercept is a good choice for another point.
Let $x = 0$.
$y = 2(0 + 1)^2 - 3$
$y = 2(1)^2 - 3$
$y = -1$
The point is (0, −1).

For any point other than the vertex, there is a corresponding point that is equidistant from the axis of symmetry. In this case, the corresponding point for (0, −1) is (−2, −1).

Plot these two additional points and complete the sketch of the parabola.



b) For the quadratic function $y = -\frac{1}{4}(x - 4)^2 + 1$, $a = -\frac{1}{4}$, $p = 4$, and $q = 1$.
The vertex is located at (4, 1).
The graph opens downward and is wider than the graph $y = x^2$.
The range is $\{y \mid y \leq 1, y \in R\}$.
The domain is $\{x \mid x \in R\}$.
The equation of the axis of symmetry is $x = 4$.

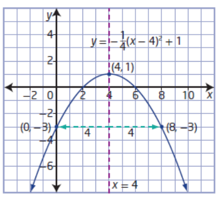
Sketch the graph of $y = -\frac{1}{4}(x - 4)^2 + 1$ by using the information from the vertex form of the function.

- Plot the vertex at (4, 1).
- Determine a point on the graph. For example, determine the y-intercept by substituting $x = 0$ into the function.

$y = -\frac{1}{4}(0 - 4)^2 + 1$
$y = -\frac{1}{4}(-4)^2 + 1$
$y = -4 + 1$
$y = -3$
The point (0, −3) is on the graph.

For any point other than the vertex, there is a corresponding point that is equidistant from the axis of symmetry. In this case, the corresponding point of (0, −3) is (7, −3).

Plot these two additional points and complete the sketch of the parabola.



How are the values of *y* affected when *a* is $-\frac{1}{4}$?

How are *p* and *q* related to the direction of the translations and location of the vertex?

How is the shape of the curve related to the value of *a*?