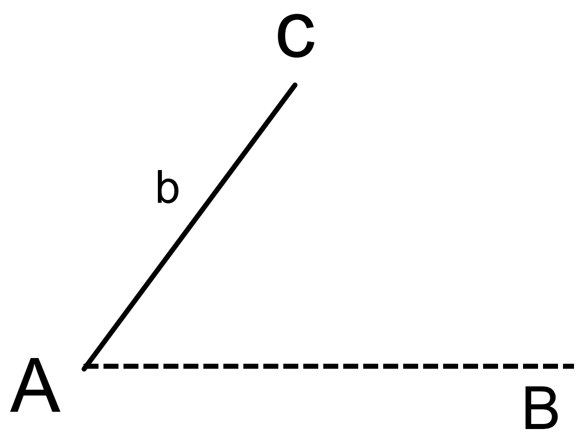
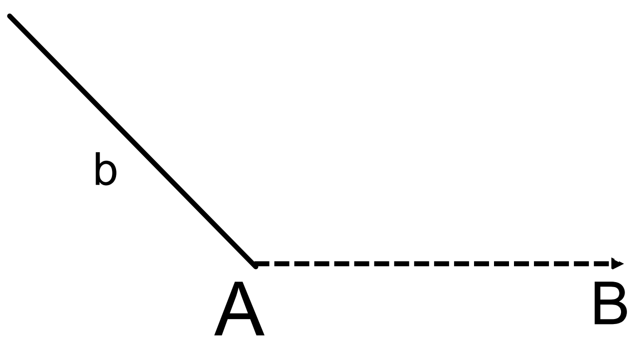
For acute Δ



For obtuse Δ



Example 3

Use the Sine Law in an Ambiguous Case

In △ABC, ∠A = 30°, a = 24 cm, and b = 42 cm. Determine the measures of the other side and angles. Round your answers to the nearest unit.

Solution

List the measures.

∠A = 30° a = 24 cm
∠B = ■ b = 42 cm
∠C = ■ c = ■

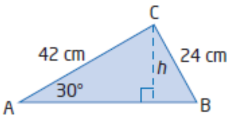
Because two sides and an angle opposite one of the sides are known, it is possible that this triangle may have two different solutions, one solution, or no solution. ∠A is acute and $a < b$, so check which condition is true.

$a < b \sin A$: no solution
$a = b \sin A$: one solution
$a > b \sin A$: two solutions

Why is the value of b sin A so important?

Sketch a possible diagram.

Where does the length of CB actually fit?



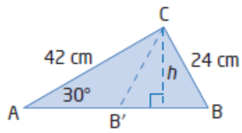
Determine the height of the triangle.

$$\sin A = \frac{h}{b}$$
$$h = b \sin A$$
$$h = 42 \sin 30°$$
$$h = 21$$

How do you know the value of sin 30°?

Since 24 > 21, the case $a > b \sin A$ occurs.

Therefore, two triangles are possible. The second solution will give an obtuse angle for ∠B.



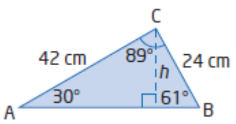
Solve for ∠B using the sine law.

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$
$$\frac{\sin B}{42} = \frac{\sin 30°}{24}$$
$$\sin B = \frac{42 \sin 30°}{24}$$
$$\angle B = \sin^{-1}\left(\frac{42 \sin 30°}{24}\right)$$
$$\angle B = 61.044...$$

To the nearest degree, ∠B = 61°.

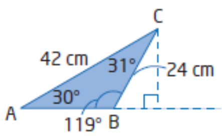
To find the second possible measure of ∠B, use 61° as the reference angle in quadrant II. Then, ∠B = 180° − 61° or ∠B = 119°.

Case 1: ∠B = 61°
∠C = 180° − (61° + 30°)
∠C = 89°



$$\frac{c}{\sin 89°} = \frac{24}{\sin 30°}$$
$$c = \frac{24 \sin 89°}{\sin 30°}$$
$$c = 47.992...$$

Case 2: ∠B = 119°
∠C = 180° − (119° + 30°)
∠C = 31°



$$\frac{c}{\sin 31°} = \frac{24}{\sin 30°}$$
$$c = \frac{24 \sin 31°}{\sin 30°}$$
$$c = 24.721...$$

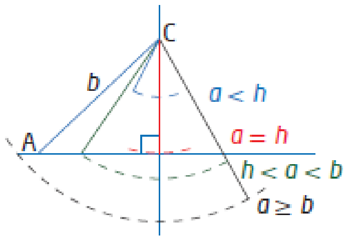
Use the sine law to determine the measure of side c in each case.

The two possible triangles are as follows:
acute △ABC: ∠A = 30°, ∠B = 61°, ∠C = 89°, a = 24 cm, b = 42 cm, c = 48 cm
obtuse △ABC: ∠A = 30°, ∠B = 119°, ∠C = 31°, a = 24 cm, b = 42 cm, c = 25 cm

Compare the ratios $\frac{a}{\sin A}$, $\frac{b}{\sin B}$, and $\frac{c}{\sin C}$ to check your answers.

- For the ambiguous case in $\triangle ABC$, when $\angle A$ is an acute angle:
  - $a \geq b$ one solution
  - $a = h$ one solution
  - $a < h$ no solution
  - $b \sin A < a < b$ two solutions

$h = b \sin A$



- For the ambiguous case in $\triangle ABC$, when $\angle A$ is an obtuse angle:
  - $a \leq b$ no solution
  - $a > b$ one solution