Chapter 4

Quadratic Formula

Write the quadratic formula and circle the discriminant.

Answer

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 1

Use the Discriminant to Determine the Nature of the Roots

Use the discriminant to determine the nature of the roots for each quadratic equation. Check by graphing.

a) $-2x^2 + 3x + 8 = 0$
b) $3x^2 - 5x = -9$
c) $\frac{1}{4}x^2 - 3x + 9 = 0$

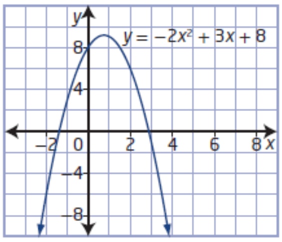
Solution

To determine the nature of the roots for each equation, substitute the corresponding values for a, b, and c into the discriminant expression, $b^2 - 4ac$.

a) For $-2x^2 + 3x + 8 = 0$, $a = -2$, $b = 3$, and $c = 8$.
$b^2 - 4ac = 3^2 - 4(-2)(8)$
$b^2 - 4ac = 9 + 64$
$b^2 - 4ac = 73$

Since the value of the discriminant is positive, there are two distinct real roots.

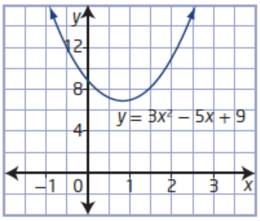
The graph of the corresponding quadratic function, $y = -2x^2 + 3x + 8$, confirms that there are two distinct x-intercepts.



b) First, rewrite $3x^2 - 5x = -9$ in the form $ax^2 + bx + c = 0$.
$3x^2 - 5x + 9 = 0$
For $3x^2 - 5x + 9 = 0$, $a = 3$, $b = -5$, and $c = 9$.
$b^2 - 4ac = (-5)^2 - 4(3)(9)$
$b^2 - 4ac = 25 - 108$
$b^2 - 4ac = -83$

Since the value of the discriminant is negative, there are no real roots. The square root of a negative number does not result in a real number.

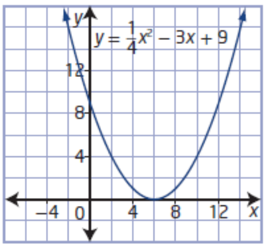
The graph of the corresponding quadratic function, $y = 3x^2 - 5x + 9$, shows that there are no x-intercepts.



c) For $\frac{1}{4}x^2 - 3x + 9 = 0$, $a = \frac{1}{4}$, $b = -3$, and $c = 9$.
$b^2 - 4ac = (-3)^2 - 4\left(\frac{1}{4}\right)(9)$
$b^2 - 4ac = 9 - 9$
$b^2 - 4ac = 0$

Since the value of the discriminant is zero, there is one distinct real root, or two equal real roots.

The graph of the corresponding quadratic function, $y = \frac{1}{4}x^2 - 3x + 9$, confirms that there is only one x-intercept because it touches the x-axis but does not cross it.



Your Turn

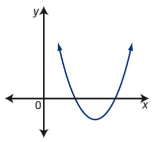
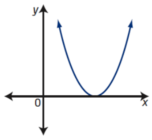
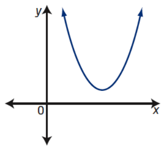
Use the discriminant to determine the nature of the roots for each quadratic equation. Check by graphing.

a) $x^2 - 5x + 4 = 0$ **b)** $3x^2 + 4x + \frac{4}{3} = 0$ **c)** $2x^2 - 8x = -9$

Chapter 4

Discriminant

What is the significance of the discriminant? Complete the following table. In the third column, sketch a graph of a quadratic function for which the condition on the left is true. The answer shows one possible graph.

	The Nature of the Roots		Graphical Representation	
$b^2 - 4ac > 0$		two real roots		
$b^2 - 4ac = 0$		one real root		
$b^2 - 4ac < 0$		no real roots		

Example 2

Use the Quadratic Formula to Solve Quadratic Equations

Use the quadratic formula to solve each quadratic equation. Express your answers to the nearest hundredth.

a) $9x^2 + 12x = -4$
b) $5x^2 - 7x - 1 = 0$

Solution

a) First, write $9x^2 + 12x = -4$ in the form $ax^2 + bx + c = 0$.
$9x^2 + 12x + 4 = 0$

For $9x^2 + 12x + 4 = 0$, $a = 9$, $b = 12$, and $c = 4$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-12 \pm \sqrt{12^2 - 4(9)(4)}}{2(9)}$$
$$x = \frac{-12 \pm \sqrt{144 - 144}}{18}$$
$$x = \frac{-12 \pm \sqrt{0}}{18}$$
$$x = \frac{-12}{18}$$
$$x = -\frac{2}{3}$$

Since the value of the discriminant is zero, there is only one distinct real root, or two equal real roots.

Check:
Substitute $x = -\frac{2}{3}$ into the original equation.

Left Side	Right Side
$9x^2 + 12x$	-4
$= 9\left(-\frac{2}{3}\right)^2 + 12\left(-\frac{2}{3}\right)$	
$= 9\left(\frac{4}{9}\right) - 8$	
$= 4 - 8$	
$= -4$	

Left Side = Right Side

How could you use technology to check your solution graphically?

The root is $-\frac{2}{3}$, or approximately -0.67.

b) For $5x^2 - 7x - 1 = 0$, $a = 5$, $b = -7$, and $c = -1$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(5)(-1)}}{2(5)}$$

$$x = \frac{7 \pm \sqrt{49 + 20}}{10}$$

$$x = \frac{7 \pm \sqrt{69}}{10}$$

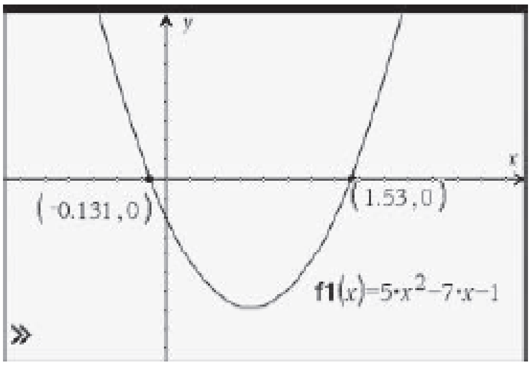
Since the value of the discriminant is positive, there are two distinct real roots.

$$x = \frac{7 + \sqrt{69}}{10} \quad \text{or} \quad x = \frac{7 - \sqrt{69}}{10}$$

$$x = 1.5306\ldots \qquad x = -0.1306\ldots$$

The roots are $\frac{7 + \sqrt{69}}{10}$ and $\frac{7 - \sqrt{69}}{10}$, or approximately 1.53 and −0.13.

Check:
The graph of the corresponding function, $y = 5x^2 - 7x - 1$, shows the zeros at approximately (−0.13, 0) and (1.53, 0).



Therefore, both solutions are correct.