Example 5

Solve for an Angle Given Its Exact Sine, Cosine, or Tangent Value

Solve for θ.

a) $\sin \theta = 0.5$, $0° \leq \theta < 360°$

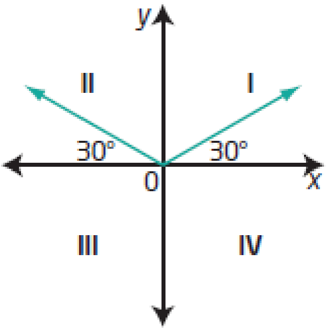
b) $\cos \theta = -\frac{\sqrt{3}}{2}$, $0° \leq \theta < 180°$

Solution

a) Since the ratio for sin θ is positive, the terminal arm lies in either quadrant I or quadrant II.

$\sin \theta_R = 0.5$
$\theta_R = 30°$

How do you know $\theta_R = 30°$?

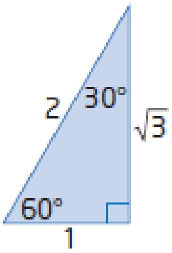


In quadrant I, $\theta = 30°$.
In quadrant II, $\theta = 180° - 30°$
$\theta = 150°$

The solution to the equation $\sin \theta = 0.5$, $0 \leq \theta < 360°$, is $\theta = 30°$ or $\theta = 150°$.

b) Since the cosine ratio is negative, the terminal arm must lie in quadrant II or quadrant III. Given the restriction $0° \leq \theta < 180°$, the terminal arm must lie in quadrant II.

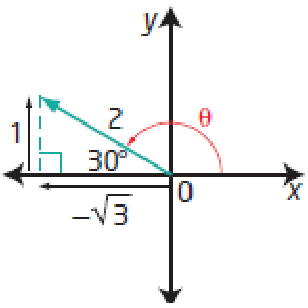
Use a 30°-60°-90° triangle to determine the reference angle, θ_R.



$\cos \theta_R = \frac{\sqrt{3}}{2}$
$\theta_R = 30°$

Using the reference angle of 30° in quadrant II, the measure of θ is $180° - 30° = 150°$.

The solution to the equation $\cos \theta = -\frac{\sqrt{3}}{2}$, $0 \leq \theta < 180°$, is $\theta = 150°$.



Solving for Angles Given Their Sine, Cosine, or Tangent

Step 1 Determine which quadrants the solution(s) will be in by looking at the sign (+ or −) of the given ratio.

Step 2 Solve for the reference angle.

Why are the trigonometric ratios for the reference angle always positive?

Step 3 Sketch the reference angle in the appropriate quadrant. Use the diagram to determine the measure of the related angle in standard position.

Example 6

Solve for an Angle Given Its Approximate Sine, Cosine, or Tangent Value

Given $\cos\theta = -0.6753$, where $0° \leq \theta < 360°$, determine the measure of θ, to the nearest tenth of a degree.

Solution

The cosine ratio is negative, so the angles in standard position lie in quadrant II and quadrant III.

Use a calculator to determine the angle that has $\cos\theta_R = 0.6753$.

$\theta_R = \cos^{-1}(0.6753)$ Why is $\cos^{-1}(0.6753)$ the reference angle?

$\theta_R \approx 47.5°$

With a reference angle of 47.5°, the measures of θ are as follows:

In quadrant II:	In quadrant III:
$\theta = 180° - 47.5°$	$\theta = 180° + 47.5°$
$\theta = 132.5°$	$\theta = 227.5°$

Key Ideas

- The primary trigonometric ratios for an angle, θ, in standard position that has a point P(x, y) on its terminal arm are $\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$, and $\tan\theta = \frac{y}{x}$, where $r = \sqrt{x^2 + y^2}$.
- The table show the signs of the primary trigonometric ratios for an angle, θ, in standard position with the terminal arm in the given quadrant.

	Quadrant			
Ratio	I	II	III	IV
sin θ	+	+	−	−
cos θ	+	−	−	+
tan θ	+	−	+	−

- If the terminal arm of an angle, θ, in standard position lies on one of the axes, θ is called a quadrantal angle. The quadrantal angles are 0°, 90°, 180°, 270°, and 360°, $0° \le \theta \le 360°$.