Solving Quadratic Equations by Factoring

Some quadratic equations that have real-number solutions can be factored easily.

The *zero product property* states that if the product of two real numbers is zero, then one or both of the numbers must be zero. This means that if $de = 0$, then at least one of d and e is 0.

The roots of a quadratic equation occur when the product of the factors is equal to zero. To solve a quadratic equation of the form $ax^2 + bx + c = 0$, $a \neq 0$, factor the expression and then set either factor equal to zero. The solutions are the roots of the equation.

For example, rewrite the quadratic equation $3x^2 - 2x - 5 = 0$ in factored form.

$$3x^2 - 2x - 5 = 0$$
$$(3x - 5)(x + 1) = 0$$

$$3x - 5 = 0 \quad \text{or} \quad x + 1 = 0$$
$$x = \frac{5}{3} \qquad\qquad x = -1$$

The roots are $\frac{5}{3}$ and -1.

Example 3

Solve Quadratic Equations by Factoring

Determine the roots of each quadratic equation. Verify your solutions.

a) $x^2 + 6x + 9 = 0$ **b)** $x^2 + 4x - 21 = 0$ **c)** $2x^2 - 9x - 5 = 0$

Solution

a) To solve $x^2 + 6x + 9 = 0$, determine the factors and then solve for x.

$$x^2 + 6x + 9 = 0$$

This is a perfect square trinomial.

$$(x + 3)(x + 3) = 0$$

$$(x + 3) = 0 \quad \text{or} \quad (x + 3) = 0$$
$$x = -3 \qquad\qquad x = -3$$

For the quadratic equation to equal 0, one of the factors must equal 0.

This equation has two equal real roots. Since both roots are equal, the roots may be viewed as one distinct real root. Check by substituting the solution into the original quadratic equation.

For $x = -3$:

Left Side	Right Side
$x^2 + 6x + 9$	0
$= (-3)^2 + 6(-3) + 9$	
$= 9 - 18 + 9$	
$= 0$	

Left Side = Right Side

The solution is correct. The roots of the equation are -3 and -3, or just -3.

b) To solve $x^2 + 4x - 21 = 0$, first determine the factors, and then solve for x.

$$x^2 + 4x - 21 = 0$$
$$(x - 3)(x + 7) = 0$$

Two integers with a product of −21 and a sum of 4 are −3 and 7.

Set each factor equal to zero and solve for x.

$x - 3 = 0$ or $x + 7 = 0$

$x = 3$ $\quad$ $x = -7$

The equation has two distinct real roots. Check by substituting each solution into the original quadratic equation.

For $x = 3$:

Left Side	Right Side
$x^2 + 4x - 21$	0
$= 3^2 + 4(3) - 21$	
$= 9 + 12 - 21$	
$= 0$	

Left Side = Right Side

For $x = -7$:

Left Side	Right Side
$x^2 + 4x - 21$	0
$= (-7)^2 + 4(-7) - 21$	
$= 49 - 28 - 21$	
$= 0$	

Left Side = Right Side

Both solutions are correct. The roots of the quadratic equation are 3 and −7.

Method 2: Factor by Grouping

Find two integers with a product of $(2)(-5) = -10$ and a sum of -9.

Factors of −10	Product	Sum
1, −10	−10	−9
2, −5	−10	−3
5, −2	−10	3
10, −1	−10	9

Write $-9x$ as $x - 10x$. Then, factor by grouping.

$$2x^2 - 9x - 5 = 0$$
$$2x^2 + x - 10x - 5 = 0$$
$$(2x^2 + x) + (-10x - 5) = 0$$
$$x(2x + 1) - 5(2x + 1) = 0$$
$$(2x + 1)(x - 5) = 0$$

Set each factor equal to zero and solve for x.

$2x + 1 = 0$ or $x - 5 = 0$

$2x = -1$ $\quad$ $x = 5$

$x = -\frac{1}{2}$

The roots are $-\frac{1}{2}$ and 5.