Chapter 4

Quadratic Equations

Answer the following questions.

a) To solve a quadratic equation of the form $ax^2 + bx + c = 0$, what quadratic function should you graph?

b) What are the roots of a quadratic equation?

c) What are the zeros of a quadratic function?

Answer

Solving/Roots & Zeros/x-intercepts

quadratic equation

- a second-degree equation with standard form $ax^2 + bx + c = 0$, where $a \neq 0$
- for example, $2x^2 + 12x + 16 = 0$

root(s) of an equation

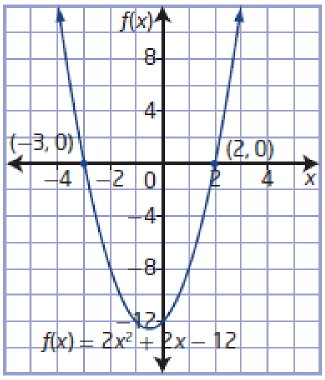
- the solution(s) to an equation

zero(s) of a function

- the value(s) of x for which $f(x) = 0$
- related to the x-intercept(s) of the graph of a function, $f(x)$

You can solve a **quadratic equation** of the form $ax^2 + bx + c = 0$ by graphing the corresponding quadratic function, $f(x) = ax^2 + bx + c$. The solutions to a quadratic equation are called the **roots** of the equation. You can find the roots of a quadratic equation by determining the x-intercepts of the graph, or the **zeros** of the corresponding quadratic function.

For example, you can solve the quadratic equation $2x^2 + 2x - 12 = 0$ by graphing the corresponding quadratic function, $f(x) = 2x^2 + 2x - 12$. The graph shows that the x-intercepts occur at $(-3, 0)$ and $(2, 0)$ and have values of -3 and 2. The zeros of the function occur when $f(x) = 0$. So, the zeros of the function are -3 and 2. Therefore, the roots of the equation are -3 and 2.

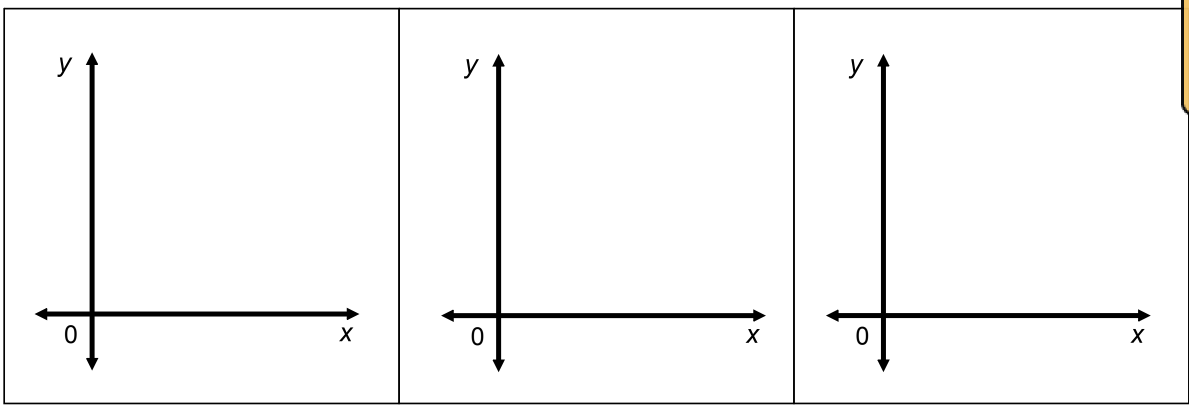


Chapter 4

Number of x-intercepts

The graph of a quadratic function can have zero, one, or two real *x*-intercepts. Sketch the graphs of quadratic functions to illustrate each of the following numbers of *x*-intercepts.

Answer



No *x*-intercept | One *x*-intercept | Two *x*-intercepts

Chapter 4

Number of Roots

Consider an equation $x^2 - 4x + k = 0$.

For what values of k does the equation have

a) one real root?

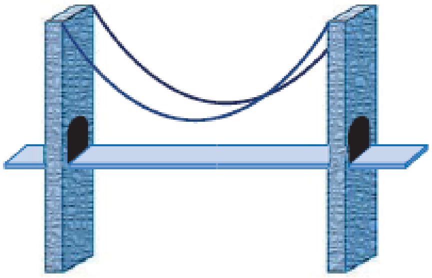
b) two distinct real roots?

c) no real roots?

Answer

Solve a Problem Involving Quadratic Equations

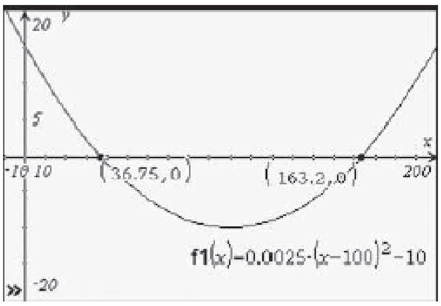
The curve of a suspension bridge cable attached between the tops of two towers can be modelled by the function $h(d) = 0.0025(d - 100)^2 - 10$, where h is the vertical distance from the top of a tower to the cable and d is the horizontal distance from the left end of the bridge, both in metres. What is the horizontal distance between the two towers? Express your answer to the nearest tenth of a metre.



Solution

At the tops of the towers, $h(d) = 0$. To determine the locations of the two towers, solve the quadratic equation $0 = 0.0025(d - 100)^2 - 10$. Graph the cable function using graphing technology. Adjust the dimensions of the graph until you see the vertex of the parabola and the x-intercepts. Use the trace or zero function to identify the x-intercepts of the graph.

The x-intercepts of the graph occur at approximately (36.8, 0) and (163.2, 0). The zeros of the function are approximately 36.8 and 163.2. Therefore, the roots of the equation are approximately 36.8 and 163.2.



What does the x-axis represent?

The first tower is located approximately 36.8 m from the left end of the bridge.

The second tower is located approximately 163.2 m from the left end of the bridge.

Subtract to determine the distance between the two towers.
$163.2 - 36.8 = 126.4$

The horizontal distance between the two towers is approximately 126.4 m.

Factoring Quadratic Expressions

To factor a trinomial of the form $ax^2 + bx + c$, where $a \neq 0$, first factor out common factors, if possible.

For example,

$$\begin{aligned}4x^2 - 2x - 12 &= 2(2x^2 - x - 6)\\ &= 2(2x^2 - 4x + 3x - 6)\\ &= 2[2x(x - 2) + 3(x - 2)]\\ &= 2(x - 2)(2x + 3)\end{aligned}$$

You can factor perfect square trinomials of the forms $(ax)^2 + 2abx + b^2$ and $(ax)^2 - 2abx + b^2$ into $(ax + b)^2$ and $(ax - b)^2$, respectively.

For example,

$$\begin{aligned}4x^2 + 12x + 9 &= (2x + 3)(2x + 3)\\ &= (2x + 3)^2\end{aligned} \qquad \begin{aligned}9x^2 - 24x + 16 &= (3x - 4)(3x - 4)\\ &= (3x - 4)^2\end{aligned}$$

You can factor a difference of squares, $(ax)^2 - (by)^2$, into $(ax - by)(ax + by)$.

For example,

$$\frac{4}{9}x^2 - 16y^2 = \left(\frac{2}{3}x - 4y\right)\left(\frac{2}{3}x + 4y\right)$$

Factoring Polynomials Having a Quadratic Pattern

You can extend the patterns established for factoring trinomials and a difference of squares to factor polynomials in quadratic form. You can factor a polynomial of the form $a(P)^2 + b(P) + c$, where P is any expression, as follows:

- Treat the expression P as a single variable, say r, by letting $r = P$.
- Factor as you have done before.
- Replace the substituted variable r with the expression P.
- Simplify the expression.

For example, in $3(x + 2)^2 - 13(x + 2) + 12$, substitute r for $x + 2$ and factor the resulting expression, $3r^2 - 13r + 12$.

$3r^2 - 13r + 12 = (3r - 4)(r - 3)$

Once the expression in r is factored, you can substitute $x + 2$ back in for r.

The resulting expression is

$$\begin{aligned}[3(x + 2) - 4](x + 2 - 3) &= (3x + 6 - 4)(x - 1)\\ &= (3x + 2)(x - 1)\end{aligned}$$

You can factor a polynomial in the form of a difference of squares, as $P^2 - Q^2 = (P - Q)(P + Q)$ where P and Q are any expressions.

For example,

$$\begin{aligned}(3x + 1)^2 - (2x - 3)^2 &= [(3x + 1) - (2x - 3)][(3x + 1) + (2x - 3)]\\ &= (3x + 1 - 2x + 3)(3x + 1 + 2x - 3)\\ &= (x + 4)(5x - 2)\end{aligned}$$

Example 1

Factor Quadratic Expressions

Factor.

a) $2x^2 - 2x - 12$

b) $\frac{1}{4}x^2 - x - 3$

c) $9x^2 - 0.64y^2$

Example 2

Factor Polynomials of Quadratic Form

Factor each polynomial.

a) $12(x + 2)^2 + 24(x + 2) + 9$
b) $9(2t + 1)^2 - 4(s - 2)^2$