Completing the Square

You can convert a quadratic function in standard form to vertex form using an algebraic process called **completing the square**. Completing the square involves adding a value to and subtracting a value from a quadratic polynomial so that it contains a perfect square trinomial. You can then rewrite this trinomial as the square of a binomial.

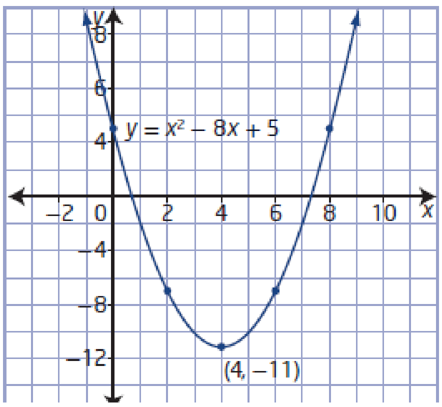
completing the square

- an algebraic process used to write a quadratic polynomial in the form $a(x - p)^2 + q$.

$y = x^2 - 8x + 5$	
$y = (x^2 - 8x) + 5$	Group the first two terms.
$y = (x^2 - 8x + 16 - 16) + 5$	Add and subtract the square of half the coefficient of the x-term.
$y = (x^2 - 8x + 16) - 16 + 5$	Group the perfect square trinomial.
$y = (x - 4)^2 - 16 + 5$	Rewrite as the square of a binomial.
$y = (x - 4)^2 - 11$	Simplify.

In the above example, both the standard form, $y = x^2 - 8x + 5$, and the vertex form, $y = (x - 4)^2 - 11$, represent the same quadratic function. You can use both forms to determine that the graph of the function will open up, since $a = 1$. However, the vertex form also reveals without graphing that the vertex is at $(4, -11)$, so this function has a minimum value of -11 when $x = 4$.

x	y
0	5
2	−7
4	−11
6	−7
8	5



Method 2: Use an Algebraic Method

For the function $y = x^2 + 6x + 5$, the value of a is 1. To complete the square,

- group the first two terms
- inside the brackets, add and subtract the square of half the coefficient of the x-term
- group the perfect square trinomial
- rewrite the perfect square trinomial as the square of a binomial
- simplify

$y = x^2 + 6x + 5$	
$y = (x^2 + 6x) + 5$	
$y = (x^2 + 6x + 9 - 9) + 5$	Why is the value 9 used here? Why is 9 also subtracted?
$y = (x^2 + 6x + 9) - 9 + 5$	Why are the first three terms grouped together?
$y = (x + 3)^2 - 9 + 5$	How is the 3 inside the brackets related to the original function? How is the 3 related to the 9 that was used earlier?
$y = (x + 3)^2 - 4$	How could you check that this is equivalent to the original expression?

Method 2: Use an Algebraic Method

To complete the square when the leading coefficient, a, is not 1,

- group the first two terms and factor out the leading coefficient
- inside the brackets, add and subtract the square of half of the coefficient of the x-term
- group the perfect square trinomial
- rewrite the perfect square trinomial as the square of a binomial
- expand the square brackets and simplify

$y = 3x^2 - 12x - 9$

$y = 3(x^2 - 4x) - 9$ — Why does 3 need to be factored from the first two terms?

$y = 3(x^2 - 4x + 4 - 4) - 9$ — Why is the value 4 used inside the brackets?

$y = 3[(x^2 - 4x + 4) - 4] - 9$

$y = 3[(x - 2)^2 - 4] - 9$

$y = 3(x - 2)^2 - 12 - 9$ — What happens to the square brackets? Why are the brackets still needed?

$y = 3(x - 2)^2 - 21$ — Why is the constant term, -21, 12 less than at the start, when only 4 was added inside the brackets?

c) Use the process of completing the square to convert to vertex form.

$y = -5x^2 - 70x$ — What happens to the x-term when a negative number is factored?

$y = -5(x^2 + 14x)$

$y = -5(x^2 + 14x + 49 - 49)$ — How does a leading coefficient that is negative affect the process? How would the result be different if it had been positive?

$y = -5[(x^2 + 14x + 49) - 49]$

$y = -5[(x + 7)^2 - 49]$

$y = -5(x + 7)^2 + 245$ — Why would algebra tiles not be suitable to use for this function?