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Chapter 3

Number of *x*-intercepts and parameters *a*, *p*, and *q*

Explore the relationship between the number of *x*-intercepts and the parameters *a*, *p*, and *q*.

Chapter 3 | Section 3.1 Example 3

Number of *x*-intercepts

Instructions

Change the values of *a*, *p*, and *q* by dragging the sliders. Observe the movement of the graph on the grid and the changes in the number of *x*-intercepts.

Chapter 3

Determine the Number of *x*-intercepts Using *a* and *q*

Complete the following table.

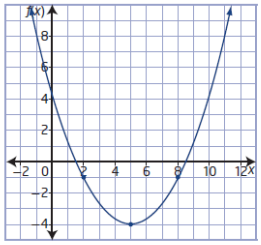
Functions	Number of *x*-intercepts	
$f(x) = -(x - 1)^2 + 8$		
$f(x) = 0.5(x - 2)^2 + 1$		
$f(x) = 2(x - 2)^2$		
$f(x) = 3x^2 - 7$		
$f(x) = -2(x + 5)^2$		
$f(x) = 3(x - 4)^2 + 3$		

Example 2

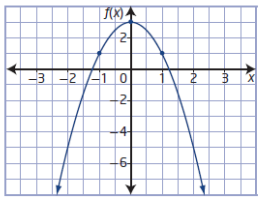
Determine a Quadratic Function in Vertex Form Given Its Graph

Determine a quadratic function in vertex form for each graph.

a)



b)



Solution

a) Method 1: Use Points and Substitution

You can determine the equation of the function using the coordinates of the vertex and one other point.

The vertex is located at $(5, -4)$, so $p = 5$ and $q = -4$. The graph opens upward, so the value of a is greater than 0.

Express the function as

$f(x) = a(x - p)^2 + q$

$f(x) = a(x - 5)^2 + (-4)$

$f(x) = a(x - 5)^2 - 4$

Choose one other point on the graph, such as $(2, -1)$. Substitute the values of x and y into the function and solve for a.

$f(x) = a(x - 5)^2 - 4$

$-1 = a(2 - 5)^2 - 4$

$-1 = a(-3)^2 - 4$

$-1 = a(9) - 4$

$-1 = 9a - 4$

$3 = 9a$

$\frac{1}{3} = a$

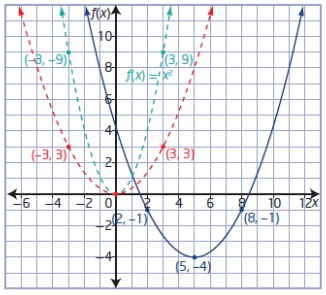
The quadratic function in vertex form is $f(x) = \frac{1}{3}(x - 5)^2 - 4$.

Method 2: Compare With the Graph of $f(x) = x^2$

The vertex is located at $(5, -4)$, so $p = 5$ and $q = -4$. The graph involves a translation of 5 units to the right and 4 units down.

The graph opens upward, so the value of a is greater than 0.

To determine the value of a, undo the translations and compare the vertical distances of points on the non-translated parabola relative to those on the graph of $f(x) = x^2$.



How are the y-coordinates of the corresponding points on the two parabolas with a vertex at (0, 0) related?

Since the vertical distances are one third as much, the value of a is $\frac{1}{3}$.

The red graph of $f(x) = \frac{1}{3}x^2$ has been stretched vertically by a factor of $\frac{1}{3}$ compared to the graph of $f(x) = x^2$.

Substitute the values $a = \frac{1}{3}$, $p = 5$, and $q = -4$ into the vertex form, $f(x) = a(x + p)^2 + q$.

The quadratic function in vertex form is $f(x) = \frac{1}{3}(x - 5)^2 - 4$.

b) You can determine the equation of the function using the coordinates of the vertex and one other point.

The vertex is located at $(0, 3)$, so $p = 0$ and $q = 3$. The graph opens downward, so the value of a is less than 0.

Express the function as

$f(x) = a(x - p)^2 + q$

$f(x) = a(x - 0)^2 + 3$

$f(x) = ax^2 + 3$

Choose one other point on the graph, such as $(1, 1)$. Substitute the values of x and y into the function and solve for a.

$f(x) = ax^2 + 3$

$1 = a(1)^2 + 3$

$1 = a + 3$

$-2 = a$

The quadratic function in vertex form is $f(x) = -2x^2 + 3$.

Example 3

Determine the Number of *x*-Intercepts Using *a* and *q*

Determine the number of *x*-intercepts for each quadratic function.

a) $f(x) = 0.8x^2 - 3$ **b)** $f(x) = 2(x - 1)^2$ **c)** $f(x) = -3(x + 2)^2 - 1$

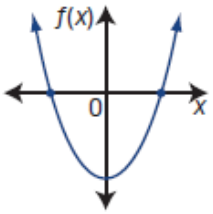
Solution

You can determine the number of *x*-intercepts if you know the location of the vertex and direction of opening. Visualize the general position and shape of the graph based on the values of *a* and *q*.

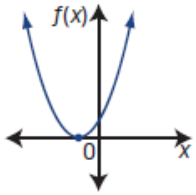
Determine the number of *x*-intercepts a quadratic function has by examining

- the value of *a* to determine if the graph opens upward or downward
- the value of *q* to determine if the vertex is above, below, or on the *x*-axis

a) $f(x) = 0.8x^2 - 3$

Value of *a*	Value of *q*	Visualize the Graph	Number of *x*-Intercepts
$a > 0$ the graph opens upward	$q < 0$ the vertex is below the *x*-axis		2 crosses the *x*-axis *twice*, since it opens *upward* from a vertex *below* the *x*-axis

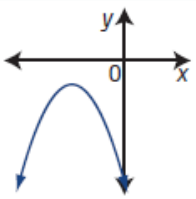
b) $f(x) = 2(x - 1)^2$

Value of *a*	Value of *q*	Visualize the Graph	Number of *x*-Intercepts
$a > 0$ the graph opens upward	$q = 0$ the vertex is on the *x*-axis		1 touches the *x*-axis *once*, since the vertex is *on* the *x*-axis

If you know that *q* is 0, does it matter what the value of *a* is?

Where on the parabola is the *x*-intercept in this case?

c) $f(x) = -3(x + 2)^2 - 1$

Value of *a*	Value of *q*	Visualize the Graph	Number of *x*-Intercepts
$a < 0$ the graph opens downward	$q < 0$ the vertex is below the *x*-axis		0 does not cross the *x*-axis, since it opens *down* from a vertex *below* the *x*-axis

Why does the value of *p* not affect the number of *x*-intercepts?

Key Ideas

- You can determine a quadratic function in vertex form if you know the coordinates of the vertex and at least one other point.
- You can determine the number of x-intercepts of the graph of a quadratic function using the value of a to determine if the graph opens upward or downward and the value of q to determine if the vertex is above, below, or on the x-axis.