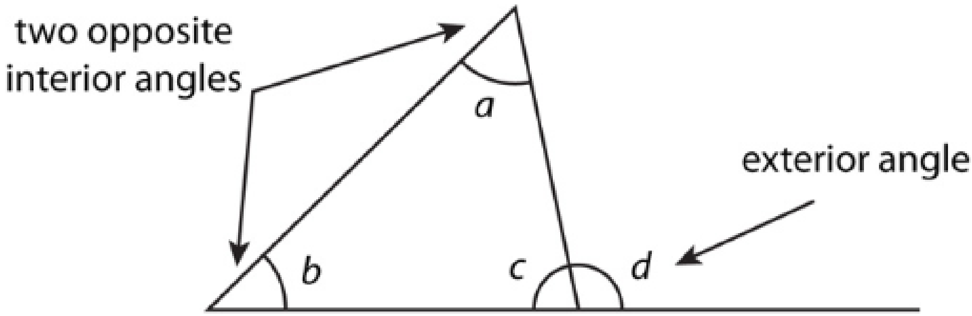
Angle Properties of Triangles

You have seen these two rules:

- The angles in a triangle add up to 180°.
- Angles on a straight line also add up to 180°.

Now, view an exploration of the **exterior angle of a triangle**. Watch how the exterior angle relates to the opposite interiors angles. Notice how the two rules, above, are used to arrive at this conjecture regarding the exterior angle of a triangle:

Conjecture: The exterior angle of any triangle is equal to the sum of the two opposite interior angles.

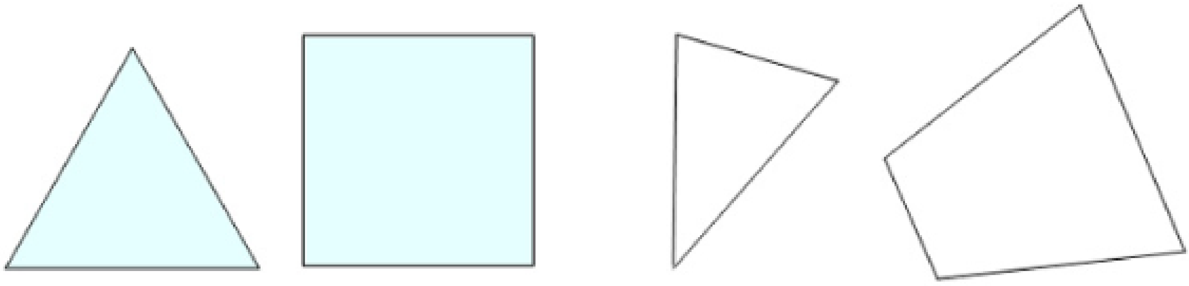


$$d = a + b$$

Angle Properties of Polygons

The simplest **polygon** is a triangle (a three-sided shape). Polygons of all types can be **regular** or **irregular**:

- **Regular polygon**: all sides of equal length, all interior angles are of equal size
- **Irregular polygon**: sides of any length, angles of any size



Regular Irregular

Identify a relationship between the number of sides in a polygon and the sum of a polygon's interior angles.

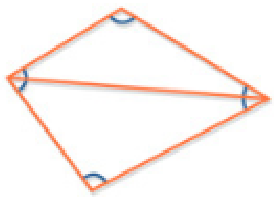
Write a formula that relates the number of sides (n) to the sum of the interior angle measures (S).

The Polygon Formula

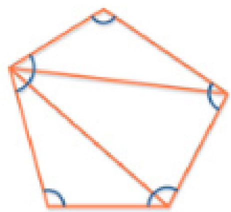
The formula developed on this page comes from dividing a polygon into triangles using full diagonals.

Begin with the rule that the interior angles of a triangle add up to 180°.

For any polygon, determine how many triangles it can be split into using full diagonals. Then, multiply the number of triangles by 180°. For each type of polygon, ask yourself how the number of triangles you were able to draw inside it relates to the number of sides on the polygon. Look for a pattern in the following examples:



This quadrilateral can be divided into two triangles, so the interior angles total: $2 \times 180 = 360°$.



This pentagon can be divided into three triangles, so the interior angles total: $3 \times 180 = 540°$.

- A hexagon can be divided into four triangles so the interior angles total ?
- A seven-sided polygon can be divided into five triangles so the interior angles total ?

The pattern can be described this way:

> For any triangulated polygon, the number of triangles will be two fewer than the number of sides.

Using this pattern, we can write a formula to calculate the sum of the interior angles of a regular polygon:

> $(n - 2) \times 180°$ where n is the number of sides of the polygon
> Sum of interior angles **$S = 180° (n - 2)$**

If you find it difficult to remember formulae, you can find the total of any polygon's interior angles by drawing and counting its triangles, as above, and then adding together as many 180° as there are triangles.

You can rearrange the formula to solve for the angles of any **regular** polygon:

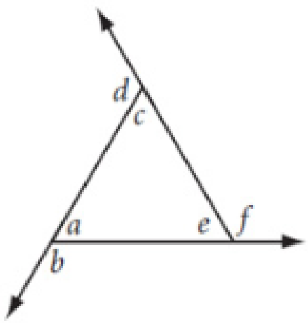
> Each angle of a regular polygon = $180° (n - 2) / n$

Interior and Exterior Angles of Regular Polygons

Now, develop a formula to calculate the sum of a regular polygon's exterior angles.

Start by considering the simplest polygon: a triangle. In this diagram, the exterior angles are: *b*, *d* and *f*. *Referring to the diagram, fill in the blanks in the flow chart.*

Enter the correct angle measurement to complete each equation as identified by the rules, theorem or principle illustrated.



1 $a + b = 180°$
angles on a straight line

2 $c + d = 180°$
angles on a straight line

3 $e + f = 180°$
angles on a straight line

4 $a + b + c + d + e + f =$ ____ °
addition

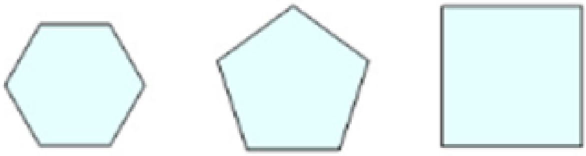
5 $a + c + e =$ ____ °
sum of a triangle theorem

6 $b + d + f =$ ____ °
subtraction

Exterior Angles of Polygons

What you should have concluded from the previous page's activity is that the exterior angles of a regular triangle add up to 360°.

What can you conclude about the total measure of exterior angles of **any** regular polygon? ***Click here*** *and view the* ***Polygons*** *animation partway down the screen for a neat illustration of this.*



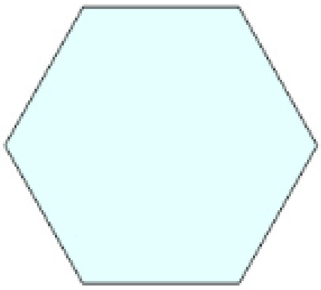
The Exterior Angles of a Polygon add up to 360°

Interior Angles of Regular Polygons

Try this:

Without measuring, determine the measure of **each interior angle** of a regular hexagon. Hint: Remember drawing interior triangles to determine the total measure of a polygon's interior angles.

Without measuring, determine the measure of **each exterior angle** of a regular hexagon.



Try this: (#1)

We know that the interior angles of a hexagon add up to 720° from the formula:

Sum of interior angles = $(n - 2) \times 180°$ where n is the number of sides

As a hexagon has six sides, **each interior angle** is equal to 720°/6 = **120°**.

We know that the exterior angles of a regular polygon always add up to 360°.

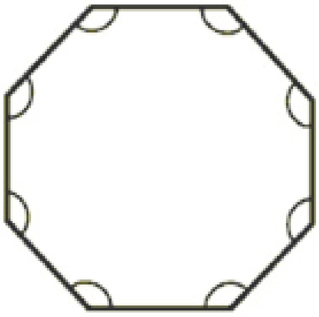
So, **each exterior angle** of a regular hexagon is 360°/6 = **60°**

Remember: The interior angle and its corresponding exterior angle always add up to 180° (for a hexagon, 120° + 60° = 180°).

Try this:

Using the same process as above, calculate:

- the measure of **each exterior angle** of a regular octagon.
- the measure of **each interior angle** of a regular octagon.



Try this: (#2)

Each exterior angle is 360°/8 = **45°**.

The sum of an interior angle and its exterior angle is 180°, so the measure of each interior angle is:

180° - 45° = 135°

The Sides of a Regular Polygon

Can you use what you know about polygons to determine how many sides a regular polygon has—without seeing that polygon—if all you know is the size on one of its angles?

You should be able to figure this out using these facts about polygons:

- The sum of an interior angle and its exterior angle is 180° (a straight line = 180°).
- The sum of the exterior angles of a polygon is 360°.

Try this:

How many sides does a regular polygon have if one of its angles = 150°?

How do you know?

SUMMARY

- The sum of the measures of the interior angles of a convex polygon with n sides can be expressed as $180^\circ (n - 2)$.
- The measure of each interior angle of a regular polygon is $\frac{180^\circ (n - 2)}{n}$
- The sum of the measures of the exterior angles of any convex polygon is 360°

Attachments

PM11-2s3-2.gsp

2s3e2 finalt.mp4