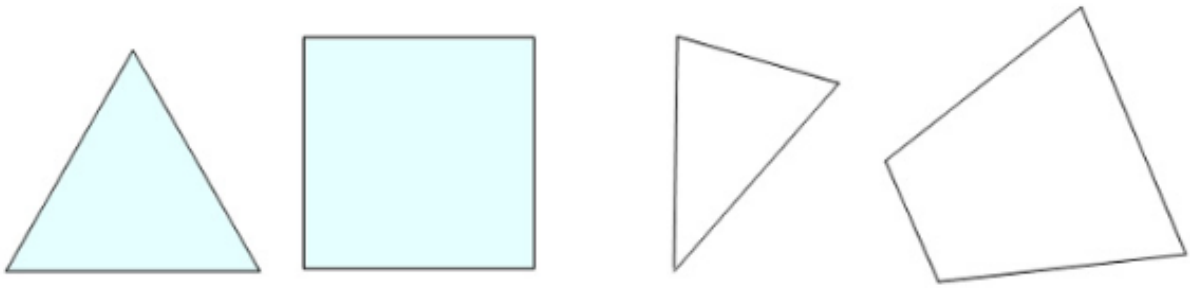
Angle Properties of Polygons

The simplest **polygon** is a triangle (a three-sided shape). Polygons of all types can be **regular** or **irregular**:

- **Regular polygon**: all sides of equal length, all interior angles are of equal size
- **Irregular polygon**: sides of any length, angles of any size



Regular **Irregular**

Identify a relationship between the number of sides in a polygon and the sum of a polygon's interior angles.

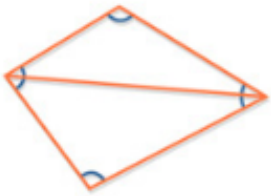
Write a formula that relates the number of sides (n) to the sum of the interior angle measures (S).

The Polygon Formula

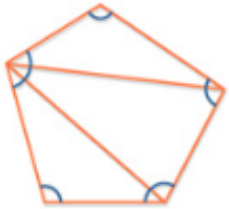
The formula developed on this page comes from dividing a polygon into triangles using full diagonals.

Begin with the rule that the interior angles of a triangle add up to 180°.

For any polygon, determine how many triangles it can be split into using full diagonals. Then, multiply the number of triangles by 180°. For each type of polygon, ask yourself how the number of triangles you were able to draw inside it relates to the number of sides on the polygon. Look for a pattern in the following examples:



This quadrilateral can be divided into two triangles, so the interior angles total: 2 × 180 = 360°.



This pentagon can be divided into three triangles, so the interior angles total: 3 × 180 = 540°.

- A hexagon can be divided into four triangles so the interior angles total ?
- A seven-sided polygon can be divided into five triangles so the interior angles total ?

The pattern can be described this way:

> For any triangulated polygon, the number of triangles will be two fewer than the number of sides.

Using this pattern, we can write a formula to calculate the sum of the interior angles of a regular polygon:

> (n - 2) × 180° where *n* is the number of sides of the polygon
> Sum of interior angles **S = 180° (n - 2)**

If you find it difficult to remember formulae, you can find the total of any polygon's interior angles by drawing and counting its triangles, as above, and then adding together as many 180° as there are triangles.

You can rearrange the formula to solve for the angles of any **regular** polygon:

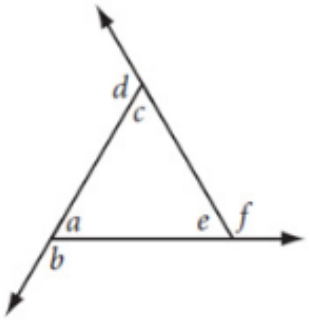
> Each angle of a regular polygon = 180° (n - 2) / *n*

Interior and Exterior Angles of Regular Polygons

Now, develop a formula to calculate the sum of a regular polygon's exterior angles.

Start by considering the simplest polygon: a triangle. In this diagram, the exterior angles are: *b*, *d* and *f*. *Referring to the diagram, fill in the blanks in the flow chart.*

Enter the correct angle measurement to complete each equation as identified by the rules, theorem or principle illustrated.



1. $a + b = 180°$ — angles on a straight line
2. $c + d = 180°$ — angles on a straight line
3. $e + f = 180°$ — angles on a straight line
4. $a + b + c + d + e + f =$ ____ ° — addition
5. $a + c + e =$ ____ ° — sum of a triangle theorem
6. $b + d + f =$ ____ ° — subtraction

Attachments

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