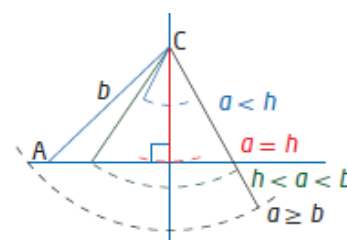


- For the ambiguous case in $\triangle ABC$, when $\angle A$ is an acute angle:

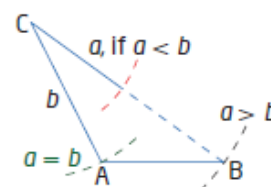
- $a \geq b$ one solution
- $a = h$ one solution
- $a < h$ no solution
- $b \sin A < a < b$ two solutions

$$h = b \sin A$$



- For the ambiguous case in $\triangle ABC$, when $\angle A$ is an obtuse angle:

- $a \leq b$ no solution
- $a > b$ one solution



Example 3

Use the Sine Law in an Ambiguous Case

In $\triangle ABC$, $\angle A = 30^\circ$, $a = 24$ cm, and $b = 42$ cm. Determine the measures of the other side and angles. Round your answers to the nearest unit.

Solution

List the measures.

$\angle A = 30^\circ$ $a = 24$ cm

$\angle B = \blacksquare$ $b = 42$ cm

$\angle C = \blacksquare$ $c = \blacksquare$

Because two sides and an angle opposite one of the sides are known, it is possible that this triangle may have two different solutions, one solution, or no solution. $\angle A$ is acute and $a < b$, so check which condition is true.

$a < b \sin A$: no solution

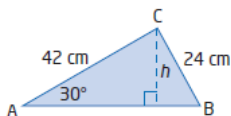
Why is the value of $b \sin A$ so important?

$a = b \sin A$: one solution

$a > b \sin A$: two solutions

Sketch a possible diagram.

Where does the length of CB actually fit?



Determine the height of the triangle.

$$\sin A = \frac{h}{b}$$

$$h = b \sin A$$

$$h = 42 \sin 30^\circ$$

How do you know the value of $\sin 30^\circ$?

$$h = 21$$

Since $24 > 21$, the case $a > b \sin A$ occurs.

Therefore, two triangles are possible. The second solution will give an obtuse angle for $\angle B$.

Solve for $\angle B$ using the sine law.

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin B}{42} = \frac{\sin 30^\circ}{24}$$

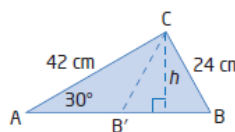
$$\sin B = \frac{42 \sin 30^\circ}{24}$$

$$\angle B = \sin^{-1} \left(\frac{42 \sin 30^\circ}{24} \right)$$

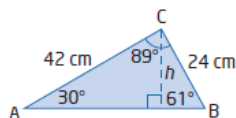
$$\angle B = 61.044\dots$$

To the nearest degree, $\angle B = 61^\circ$.

To find the second possible measure of $\angle B$, use 61° as the reference angle in quadrant II. Then, $\angle B = 180^\circ - 61^\circ$ or $\angle B = 119^\circ$.



Case 1: $\angle B = 61^\circ$
 $\angle C = 180^\circ - (61^\circ + 30^\circ)$
 $\angle C = 89^\circ$

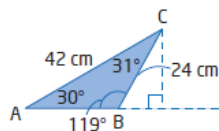


$$\frac{c}{\sin 89^\circ} = \frac{24}{\sin 30^\circ}$$

$$c = \frac{24 \sin 89^\circ}{\sin 30^\circ}$$

$$c = 47.992\dots$$

Case 2: $\angle B = 119^\circ$
 $\angle C = 180^\circ - (119^\circ + 30^\circ)$
 $\angle C = 31^\circ$



$$\frac{c}{\sin 31^\circ} = \frac{24}{\sin 30^\circ}$$

$$c = \frac{24 \sin 31^\circ}{\sin 30^\circ}$$

$$c = 24.721\dots$$

Use the sine law to determine the measure of side c in each case.

The two possible triangles are as follows:

acute $\triangle ABC$: $\angle A = 30^\circ$, $\angle B = 61^\circ$, $\angle C = 89^\circ$,
 $a = 24$ cm, $b = 42$ cm, $c = 48$ cm

obtuse $\triangle ABC$: $\angle A = 30^\circ$, $\angle B = 119^\circ$, $\angle C = 31^\circ$,
 $a = 24$ cm, $b = 42$ cm, $c = 25$ cm

Compare the ratios $\frac{a}{\sin A}$, $\frac{b}{\sin B}$, and $\frac{c}{\sin C}$ to check your answers.