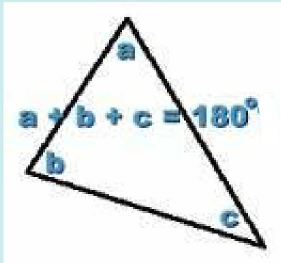
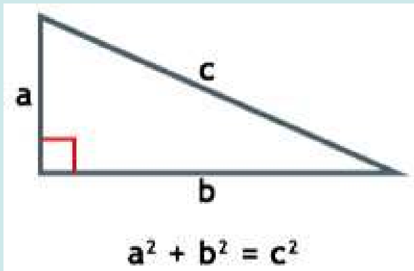
TRIANGLES....

We know that......

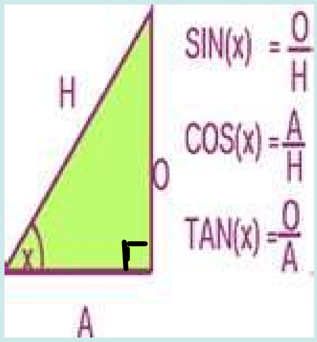
- to find an unknown angle given two angles in any triangle we use the Angle Sum of a Triangle Theorem (sum = 180^o)



- to find an unknown side given two sides in a right-triangle we use Pythagorean Theorem (hyp^2 = side^2 + side^2)



- to find sides or angles given one side and one angle in a right triangle we use Primary Trigonometric Ratios (sin, cos, tan)

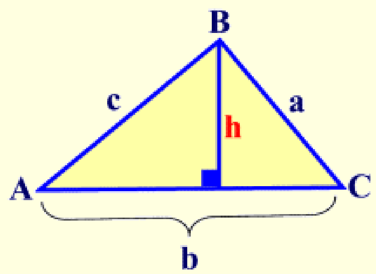


But what if we want to find sides or angles given one side and one angle in an oblique (non-right) triangle?

One possibility is to use the Law of Sines

Law of Sines

Triangle *ABC* at the right does not contain a right angle. A perpendicular is dropped from vertex *B*. It can now be observed that:



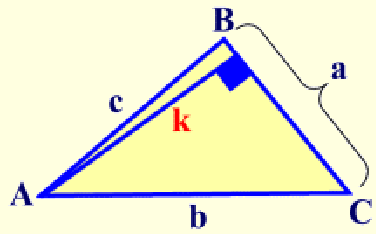
$$\sin\measuredangle A = \frac{h}{c} \quad \Rightarrow \quad h = c\sin\measuredangle A$$

$$\sin\measuredangle C = \frac{h}{a} \quad \Rightarrow \quad h = a\sin\measuredangle C$$

$$h = c\sin\measuredangle A = a\sin\measuredangle C$$

Since $c\sin\measuredangle A = a\sin\measuredangle C$, we have $\dfrac{\sin A}{a} = \dfrac{\sin C}{c}$.

Now, drop a perpendicular from vertex A. It can be observed that:



$$\sin\measuredangle B = \frac{k}{c} \quad \Rightarrow \quad k = c\sin\measuredangle B$$

$$\sin\measuredangle C = \frac{k}{b} \quad \Rightarrow \quad k = b\sin\measuredangle C$$

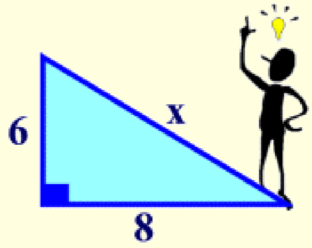
$$k = c\sin\measuredangle B = b\sin\measuredangle C$$

Since $c\sin\measuredangle B = b\sin\measuredangle C$, we have $\dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

$$\boxed{\text{Law of Sines: } \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}}$$

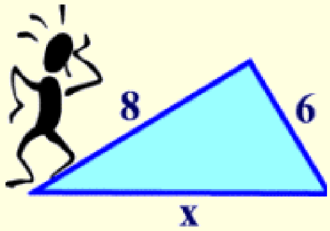
True for ALL triangles!

When you see the triangle below on the left and someone asks you to find the value of *x*, you immediately know how to proceed. You call upon your friend the Pythagorean Theorem because you see the right angle.

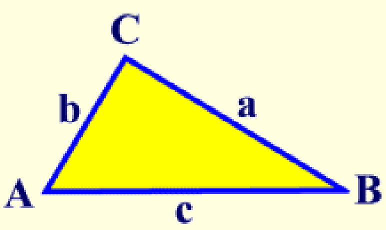


But what do you do when you see the triangle on the right? There is no indication of a right angle.

Now, with our knowledge of trigonometry, we are armed to attack any of these perplexing problems!



Let's see how to apply trigonometry to working in triangles which do not contain a right angle.



In this diagram, notice how the triangle is labeled. The capital letters for the vertices are repeated in small case on the side opposite the corresponding vertex.

side a is opposite <A
side b is opposite <B
side c is opposite <C

working together as partners!

The ratios of each side to the sine of its "partner" are equal to each other.

If a problem refers to 2 sides and 2 angles, use the Law of Sines.

Law of Sines

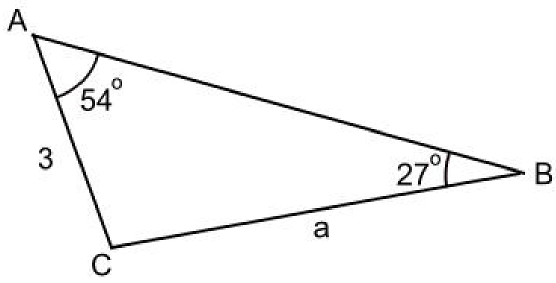
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

or

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

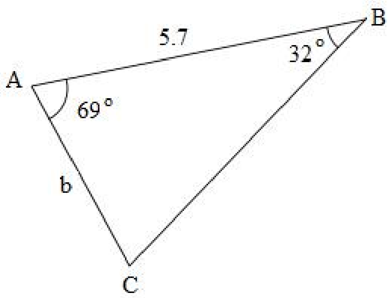
These ratios, in pairs, are applied to solving problems. You never need to use all three ratios at the same time. Mix and match the ratios to correspond with the letters you need. Remember

What is the length of side a?



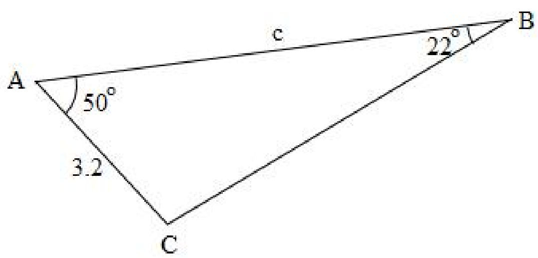
A 6	B 5.346
C 3.663	D 5.683

What is the length of side b?



A 10.56	B 5.42
C 3.24	D 3.08

What is the length of side c?



A 9.16	B 8.12
C 6.54	D 1.26