1.3.2 In Class or Homework Exercise

1. A car accelerates from 40. km/h up to 80. km/h in 5.0 s. What was the acceleration in m/s^2, and how far did it travel in this time? Assume constant acceleration.

$$\vec{v}_i = 40.km/h = 11.1m/s$$
$$\vec{v}_f = 80.km/h = 22.2m/s$$
$$t = 5.0s$$
$$\vec{a} = ?$$
$$\Delta\vec{d} = ?$$

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t} = \frac{22.2 - 11.1}{5.0} = \boxed{2.2m/s^2}$$

$$\Delta\vec{d} = \frac{\vec{v}_i + \vec{v}_f}{2}t = \frac{11.1 + 22.2}{2}(5.0) = \boxed{83m}$$

2. A car decelerates from a speed of 30.0 *m/s* to rest in 6.00 s. How far did it travel in that time?

$$\vec{v}_i = 30.0m/s$$
$$\vec{v}_f = 0$$
$$t = 6.00s$$
$$\Delta\vec{d} = ?$$

$$\Delta\vec{d} = \frac{\vec{v}_i + \vec{v}_f}{2}t = \frac{30.0 + 0}{2}(6.00) = \boxed{90.0m}$$

3. A student is rushing to get to physics class on time, accelerating at a rate of 0.90 m/s^2 (from rest). If the student starts at the office (20.0 m away), how long does it take to get to class?

$$\vec{v}_i = 0$$
$$\vec{a} = 0.90m/s^2$$
$$\Delta\vec{d} = 20.0$$
$$t = ?$$

$$\Delta\vec{d} = \vec{v}_i t + \tfrac{1}{2}\vec{a}t^2$$
$$20.0 = 0 + \tfrac{1}{2}(0.90)t^2$$
$$t = \boxed{6.7s}$$

4. A person throws a ball with a speed of 120. km/h. Calculate the acceleration, if the person accelerates the ball through a distance of 3.50 m.

$$\vec{v}_i = 0$$
$$\vec{v}_f = 120.km/h = 33.3m/s$$
$$\Delta\vec{d} = 3.50m$$
$$\vec{a} = ?$$

$$\Delta\vec{d} = \frac{v_f^2 - v_i^2}{2\vec{a}}$$
$$3.50 = \frac{33.3^2 - 0}{2\vec{a}}$$
$$\vec{a} = \boxed{159m/s^2}$$

5. A car moves at 12 *m/s* and coasts up a hill with a uniform acceleration of -1.6 m/s^2.

   a. What is the displacement after 6.0 *s*?

$$\vec{v}_i = 12m/s$$
$$\vec{a} = -1.6m/s^2$$
$$t = 6.0s$$
$$\Delta\vec{d} = ?$$

$$\Delta\vec{d} = \vec{v}_i t + \tfrac{1}{2}\vec{a}t^2$$
$$= (12)(6.0) + \tfrac{1}{2}(-1.6)(6.0)^2$$
$$= \boxed{43m}$$

b. What is the displacement after 9.0 *s*?

$$\vec{v}_i = 12m/s$$
$$\vec{a} = -1.6m/s^2$$
$$t = 9.0s$$
$$\Delta\vec{d} = ?$$

$$\Delta\vec{d} = \vec{v}_i t + \tfrac{1}{2}\vec{a}t^2$$
$$= (12)(9.0) + \tfrac{1}{2}(-1.6)(9.0)^2$$
$$= \boxed{43m}$$

c. Notice anything surprising? Try to explain.

The car has the exact same displacement. After 6.0 s, the car continued to go up the hill while slowing down. At some point, it stopped and started coming back down the hill; at 9.0 s, it happened to be at the same place as it was at 6.0 s. This is a good example that these equations provide displacement, not distance.

6. A car accelerates at a constant rate from 15 *m*/*s* to 25 *m*/*s* while it travels 125 *m*. How long does this take?

$$\vec{v}_i = 15m/s$$
$$\vec{v}_f = 25m/s$$
$$\Delta\vec{d} = 125m$$
$$t = ?$$

$$\Delta\vec{d} = \frac{\vec{v}_i + \vec{v}_f}{2}t$$
$$125 = \frac{15+25}{2}t$$
$$t = \boxed{6.3s}$$

7. Suppose a planner is designing an airport for small planes. One kind of airplane that might use this airfield must reach a speed of 200. km/h in order to take off, and can accelerate at 12.0 m/s^2. If the runway is 100. m long, can this airplane reach the proper speed to take off? If not, how long must the runway be?

We will assume that the runway is not long enough, and attempt to calculate how long it must be.

$$\vec{v}_i = 0$$
$$\vec{v}_f = 200.km/h = 55.6m/s$$
$$\vec{a} = 12.0m/s^2$$
$$\Delta\vec{d} = ?$$

$$\Delta\vec{d} = \frac{v_f^2 - v_i^2}{2\vec{a}}$$
$$= \frac{55.6^2 - 0}{2(12.0)}$$
$$= \boxed{129m}$$

The runway is not long enough; it must be at least 129 m long.

8. At high speeds a car can accelerate at 0.500 m/s^2. How long will it take this car to accelerate from 80.0 km/h to 120.0 km/h?

$$\vec{v}_i = 80.0 km/h = 22.2 m/s$$
$$\vec{v}_f = 120.0 km/h = 33.3 m/s$$
$$\vec{a} = 0.500 m/s^2$$

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t}$$
$$0.500 = \frac{33.3 - 22.2}{t}$$
$$t = \boxed{22.2 s}$$

9. A car accelerates from 20.0 km/h to 80.0 km/h in 8.00 s.

a. What is the acceleration in m/s^2?

$$\vec{v}_i = 20.0 km/h = 5.56 m/s$$
$$\vec{v}_f = 80.0 km/h = 22.2 m/s$$
$$t = 8.00 s$$

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t} = \frac{22.2 - 5.56}{8.00} = \boxed{2.08 m/s^2}$$

b. How far did the car travel in this time?

$$\Delta\vec{d} = \frac{\vec{v}_i + \vec{v}_f}{2} t = \frac{5.56 + 22.2}{2}(8.00) = \boxed{111 m}$$

10. A car traveling 80. km/h decelerates at -1.5 m/s^2. Find

a. the distance it goes before it stops

$$\vec{v}_i = 80. km/h = 22.2 m/s$$
$$\vec{v}_f = 0$$
$$\vec{a} = -1.5 m/s^2$$
$$\Delta\vec{d} = ?$$

$$\Delta\vec{d} = \frac{v_f^2 - v_i^2}{2\vec{a}} = \frac{0 - 22.2^2}{2(-1.5)} = \boxed{160 m}$$

b. the time it takes to stop

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t}$$
$$-1.5 = \frac{0 - 22.2}{t}$$
$$t = \boxed{15 s}$$

c. the distance it travels during the 1*st* and 3*rd* seconds

First Second

$$\vec{v}_i = 80.km/h = 22.2m/s$$
$$t = 1.0s$$
$$\vec{a} = -1.5m/s^2$$
$$\Delta\vec{d}_1 = ?$$

$$\Delta\vec{d}_1 = \vec{v}_i t + \tfrac{1}{2}\vec{a}t^2$$
$$= (22.2)(1.0) + \tfrac{1}{2}(-1.5)(1.0)^2$$
$$= \boxed{21m}$$

Third Second

If we use t=3.0 s, this will tell us how far it went in the first 3 seconds:

$$\vec{v}_i = 80.km/h = 22.2m/s$$
$$t = 3.0s$$
$$\vec{a} = -1.5m/s^2$$
$$\Delta\vec{d}_3 = ?$$

$$\Delta\vec{d}_3 = \vec{v}_i t + \tfrac{1}{2}\vec{a}t^2$$
$$= (22.2)(3.0) + \tfrac{1}{2}(-1.5)(3.0)^2$$
$$= 59.85m$$

Using t=2.0 s will tell us how far it went in the first 2 seconds:

$$\vec{v}_i = 80.km/h = 22.2m/s$$
$$t = 2.0s$$
$$\vec{a} = -1.5m/s^2$$
$$\Delta\vec{d}_2 = ?$$

$$\Delta\vec{d}_2 = \vec{v}_i t + \tfrac{1}{2}\vec{a}t^2$$
$$= (22.2)(2.0) + \tfrac{1}{2}(-1.5)(2.0)^2$$
$$= 41.4m$$

So the distance traveled in the third second (between t=2.0 and t=3.0s) is

$$\Delta d = 59.85 - 41.4$$
$$= 18.45m$$
$$= \boxed{18m}$$

11. How far does a car travel before stopping if it has an initial speed of 80.0 km/h and a human reaction time of 1.0 s and an acceleration of -4.0 m/s^2?

The problem must be broken up into 2 parts, the reaction time (during which the velocity is constant) and the time while slowing down.

Reaction Time

$$\vec{v} = 80.0km/h = 22.2m/s$$
$$t = 1.0s$$
$$\Delta\vec{d}_1 = ?$$

$$\vec{v} = \frac{\Delta\vec{d}_1}{t}$$
$$22.2 = \frac{\Delta\vec{d}_1}{1.0}$$
$$\Delta\vec{d}_1 = 22m$$

Slowing Down

$\vec{v}_i = 80.0km/h = 22.2m/s$

$\vec{v}_f = 0$

$\vec{a} = -4.0m/s^2$

$\Delta\vec{d}_2 = ?$

$$\Delta\vec{d}_2 = \frac{v_f^2 - v_i^2}{2\vec{a}} = \frac{0 - 22.2^2}{2(-4.0)} = 62m$$

$$\Delta\vec{d}_t = \Delta\vec{d}_1 + \Delta\vec{d}_2 = 22 + 62 = \boxed{84m}$$

The total displacement of the car is 84 m.

12. You are looking at new cars and a foolish salesman lets you take one for a test drive. While driving, you accelerate from rest at 2.2 m/s^2 for 5.2 s. You then maintain this speed for 3.5 s, then accelerate at 3.8 m/s^2 for 4.5 s.

    a. How far have you travelled?

Part 1

$\vec{v}_{1i} = 0$

$\vec{a}_1 = 2.2m/s^2$

$t_1 = 5.2s$

$\Delta\vec{d}_1 = ?$

$$\Delta\vec{d}_1 = \vec{v}_{1i}t_1 + \tfrac{1}{2}\vec{a}_1 t_1^2 = 0 + \tfrac{1}{2}(2.2)(5.2)^2 = 29.7m$$

$$\vec{a}_1 = \frac{\vec{v}_{1f} - \vec{v}_{1i}}{t_1}$$

$$2.2 = \frac{\vec{v}_{1f} - 0}{5.2}$$

$$\vec{v}_{1f} = 11.4m/s$$

Part 2

$\vec{v}_2 = \vec{v}_{1f} = 11.4m/s$

$t_2 = 3.5s$

$$\vec{v}_2 = \frac{\Delta\vec{d}_2}{t_2}$$

$$11.4 = \frac{\Delta\vec{d}_2}{3.5}$$

$$\Delta\vec{d}_2 = 39.9m$$

Part 3

$\vec{v}_{3i} = 11.4m/s$

$\vec{a}_3 = 3.8m/s^2$

$t_3 = 4.5s$

$\Delta\vec{d}_3 = ?$

$$\Delta\vec{d}_3 = \vec{v}_{3i}t_3 + \tfrac{1}{2}\vec{a}_3 t_3^2 = (11.4)(4.5) + \tfrac{1}{2}(3.8)(4.5)^2 = 89.8m$$

$$\begin{aligned}\Delta\vec{d}_t &= \Delta\vec{d}_1 + \Delta\vec{d}_2 + \Delta\vec{d}_3 \\ &= 29.7 + 39.9 + 89.8 \\ &= 159.4m \\ &= \boxed{160m}\end{aligned}$$

b. What was your average speed?

$\Delta d_t = 160m$

$t_t = 13.2s$

$v = ?$

$$\begin{aligned} v &= \frac{\Delta d}{t} \\ &= \frac{160}{13.2} \\ &= \boxed{12m/s}\end{aligned}$$

13. A 100.0 m long train accelerates uniformly from rest. If the front of the train passes a worker, 150.0 m down the track, with a speed of 30.0 m/s, then find the speed of the train when the last car passes the stationary worker.

Since the acceleration is the same throughout the entire problem, if we can find this in part 1 then it can be used in part 2.

Part 1

$\vec{v}_i = 0$

$\vec{v}_f = 30.0m/s$

$\Delta\vec{d} = 150.0$

$\vec{a} = ?$

$$\begin{aligned}\Delta\vec{d} &= \frac{v_f^2 - v_i^2}{2\vec{a}} \\ 150.0 &= \frac{30.0^2 - 0}{2\vec{a}} \\ \vec{a} &= 3.00m/s^2\end{aligned}$$

Part 2

$\vec{v}_i = 30.0m/s$

$\vec{a} = 3.00m/s^2$

$\Delta\vec{d} = 100.0m$

$\vec{v}_f = ?$

$$\begin{aligned}\Delta\vec{d} &= \frac{v_f^2 - v_i^2}{2\vec{a}} \\ 100.0 &= \frac{v_f^2 - 30.0^2}{2(3.00)} \\ v_f &= \boxed{38.7m/s}\end{aligned}$$

14. You are driving your car at 50.0 km/h. You are 30.0 m away from an intersection when the light turns amber. The amber light lasts for only 2.00 s. The intersection is 12.0 m wide, and your car can decelerate at 6.00 m/s^2.

a. Are you going to be able to stop?

The important criterion for the car being able to stop is that it not enter the intersection; therefore, it must be able to stop in a distance less than 30.0m. The time of 2.00 s is not important, nor is the width of the intersection.

$$\vec{v}_i = 50.0km/h = 13.9m/s$$
$$\vec{v}_f = 0$$
$$\vec{a} = -6.00m/s^2$$

$$\begin{aligned}\Delta\vec{d} &= \frac{v_f^2 - v_i^2}{2\vec{a}} \\ &= \frac{0 - 13.9^2}{2(-6.00)} \\ &= \boxed{16.1m}\end{aligned}$$

Since $16.1m < 30.0m$, the car is able to stop.

b. Your car can accelerate from 50.0 km/h to 70.0 km/h in 7.00 s. Could you make it to the other side of the intersection before the light turned red, if you accelerated? (Ignore reaction times for this problem)

First, it is necessary to find the acceleration.

$$\vec{v}_i = 50.0km/h = 13.9m/s$$
$$\vec{v}_f = 70.0km/h = 19.4m/s$$
$$t = 7.00s$$

$$\begin{aligned}\vec{a} &= \frac{\vec{v}_f - \vec{v}_i}{t} \\ &= \frac{19.4 - 13.9}{7.00} \\ &= 0.786m/s^2\end{aligned}$$

The important criterion for the car being able to make it through the intersection is for the car to get to the other side of the intersection in the 2.00 s before the light turns red.

$$t = 2.00s$$
$$\vec{v}_i = 50.0km/h = 13.9m/s$$
$$\vec{a} = 0.786m/s^2$$
$$\Delta\vec{d} = ?$$

$$\begin{aligned}\Delta\vec{d} &= \vec{v}_i t + \tfrac{1}{2}\vec{a}t^2 \\ &= (13.9)(2.00) + \tfrac{1}{2}(0.786)(2.00)^2 \\ &= \boxed{29.4m}\end{aligned}$$

Since $29.4m < 42.0m$, you would not be able to make it to the other side of the intersection (you would in fact just be entering the intersection when the light turns red).

15. A red car goes through a stop light at a speed of 60.0 *km/h*. A police car (at rest) starts chasing the car with an acceleration of 4.5 m/s^2. How far away from the intersection does the police car catch the red car if:

a. the police car leaves the intersection at the same time

Red Car

$$\vec{v}_r = 60.0km/h = 16.7m/s$$
$$t_r = t$$
$$\Delta\vec{d}_r = x$$

Police Car

$$\vec{v}_{pi} = 0$$
$$\vec{a}_p = 4.5m/s^2$$
$$\Delta\vec{d}_p = x$$
$$t_p = t$$

Setting up an equation for each vehicle, we get

$$\vec{v}_r = \frac{\Delta\vec{d}_r}{t_r} \qquad \text{and} \qquad \Delta\vec{d}_p = \vec{v}_{pi}t_p + \tfrac{1}{2}\vec{a}_p t_p{}^2$$

$$16.7 = \frac{x}{t} \qquad\qquad x = 0 + \tfrac{1}{2}(4.5)t^2$$

$$x = 2.25t^2$$

Solving this system of equations gives

$$16.7 = \frac{2.25t^2}{t} \qquad \text{and} \qquad \Delta\vec{d}_p = \vec{v}_{pi}t_p + \tfrac{1}{2}\vec{a}_p t_p{}^2$$

$$t = 7.42s \qquad\qquad = 0 + \tfrac{1}{2}(4.5)(7.42)^2$$

$$= \boxed{120m}$$

b. the police car leaves the intersection 3.0 *s* later

Red Car

$\vec{v}_r = 60.0km/h = 16.7m/s$

$t_r = t + 3.0$

$\Delta\vec{d}_r = x$

Police Car

$\vec{v}_{pi} = 0$

$\vec{a}_p = 4.5m/s^2$

$\Delta\vec{d}_p = x$

$t_p = t$

Setting up an equation for each vehicle, we get

$$\vec{v}_r = \frac{\Delta\vec{d}_r}{t_r} \qquad \text{and} \qquad \Delta\vec{d}_p = \vec{v}_{pi}t_p + \tfrac{1}{2}\vec{a}_p t_p{}^2$$

$$16.7 = \frac{x}{t+3.0} \qquad\qquad x = 0 + \tfrac{1}{2}(4.5)t^2$$

$$x = 2.25t^2$$

Solving this system of equations gives

$$16.7 = \frac{2.25t^2}{t+3.0}$$

$$0 = 2.25t^2 - 16.7t - 50.1$$

$$t = 9.7s \textbf{ or } -2.3s \quad \textbf{(using the quadratic formula)}$$

Since we can't have a negative time,

$$\Delta\vec{d}_p = \vec{v}_{pi}t_p + \tfrac{1}{2}\vec{a}_p t_p{}^2$$

$$= 0 + \tfrac{1}{2}(4.5)(9.7)^2$$

$$= \boxed{210m}$$

16. Car A has a headstart of 740. m ahead of car B. Car A is travelling at a constant 50.0 km/h while car B starts from rest and accelerates at a constant 1.22 m/s^2. How long will it take car B to catch car A and how far will car B have travelled?

Car A

$\vec{v}_A = 50.0 km/h = 13.9 m/s$

$t_A = t$

$\Delta\vec{d}_A = x$

Car B

$\vec{v}_{Bi} = 0$

$\vec{a}_B = 1.22 m/s^2$

$t_B = t$

$\Delta\vec{d}_B = x + 740.$

Setting up an equation for each vehicle, we get

$$\vec{v}_A = \frac{\Delta\vec{d}_A}{t_A}$$
$$13.9 = \frac{x}{t}$$

and

$$\Delta\vec{d}_B = \vec{v}_{Bi}t_B + \tfrac{1}{2}\vec{a}_B t_B{}^2$$
$$x + 740. = 0 + \tfrac{1}{2}(1.22)t^2$$
$$x = 0.61t^2 - 740.$$

Solving this system of equations gives

$$13.9 = \frac{0.61t^2 - 740.}{t}$$
$$0 = 0.61t^2 - 13.9t - 740.$$
$$t = 48.0s \textbf{ or } -25.3s \textbf{ (using the quadratic formula)}$$

$$\Delta\vec{d}_B = \vec{v}_{Bi}t_B + \tfrac{1}{2}\vec{a}_B t_B{}^2$$
$$= 0 + \tfrac{1}{2}(1.22)(48.0)^2$$
$$= \boxed{1410m}$$