3.3.1 In Class or Homework Exercise

1. An object is released from a height of 11 *m*. How fast is it travelling at a height of 6.0 *m*?

   Since the object is "released", the initial speed is zero so the initial kinetic energy is zero.

   $h_i = 11m$
   $h_f = 6.0m$
   $v_i = 0$
   $v_f = ?$

   $$E_i = E_f$$
   $$E_{pi} = E_{pf} + E_{kf}$$
   $$\cancel{m}gh_i = \cancel{m}gh_f + \tfrac{1}{2}\cancel{m}v_f^2$$
   $$(9.80)(11) = (9.80)(6.0) + \tfrac{1}{2}v_f^2$$
   $$v_f = \boxed{9.9m/s}$$

2. An object slides down a frictionless 33.0^o incline whose vertical height is 14.0 *cm*. How fast is it going when it reaches the bottom?

   If we use the bottom of the ramp as the reference level, we have only potential energy at the top and only kinetic energy at the bottom:

   $h_i = 14.0cm = 0.14m$
   $h_f = 0$
   $v_i = 0$
   $v_f = ?$

   $$E_i = E_f$$
   $$E_{pi} = E_{kf}$$
   $$\cancel{m}gh_i = \tfrac{1}{2}\cancel{m}v_f^2$$
   $$(9.80)(0.14) = \tfrac{1}{2}v_f^2$$
   $$v_f = \boxed{1.7m/s}$$

   Notice that the angle was not relevant and did not need to be used.

3. A bike rider approaches a hill with a speed of 8.5 *m/s*. The total mass of the bike and rider is 85 *kg*. Assuming that there is no friction, at what height will the bike come to a stop?

   We will use the bottom of the hill as the reference level.

   Initial: bike has kinetic energy because it is moving, but no potential energy because it is at the reference level

   Final: Has potential energy because it is above the reference level, but no kinetic energy because it is not moving

   $h_i = 0$
   $h_f = ?$
   $v_i = 8.5m/s$
   $v_f = 0$

   $$E_i = E_f$$
   $$E_{ki} = E_{pf}$$
   $$\tfrac{1}{2}\cancel{m}v_i^2 = \cancel{m}gh_f$$
   $$\tfrac{1}{2}(8.5)^2 = (9.80)h_f$$
   $$h_f = \boxed{3.7m}$$

   Notice that the mass cancelled out, so it was not needed.

4. A skier starts from rest at the top of a 45.0 *m* high hill, skis down a 30^o incline into a valley, and continues up a 40.0 *m* high hill.
   a. How fast is the skier moving at the bottom of the valley?
      We will use the top of the 45.0 m high hill as our initial point and the valley as our final point. The valley will be our reference level.
      Initial: there is no kinetic energy since the skier is starting from rest, there is potential energy since he is above the reference level
      Final: There is kinetic energy since the skier is moving, there is no potential energy since the skier is at the reference level

$$h_i = 45.0m \quad h_f = 0 \quad v_i = 0 \quad v_f = ?$$

$$E_i = E_f$$
$$E_{pi} = E_{kf}$$
$$mgh_i = \tfrac{1}{2}mv_f^2$$
$$(9.80)(45.0) = \tfrac{1}{2}v_f^2$$
$$v_f = \boxed{29.7 m/s}$$

   b. What is the skier's speed at the top of the next hill?
      We will use the top of the 45.0 m high hill as our initial point and the top of the 40.0m high hill as our final point. The valley will be our reference level.
      Initial: there is no kinetic energy since the skier is starting from rest, there is potential energy since he is above the reference level
      Final: There is kinetic energy since the skier is moving, there is also potential energy since the skier is above the reference level

$$h_i = 45.0m \quad h_f = 40.0 \quad v_i = 0 \quad v_f = ?$$

$$E_i = E_f$$
$$E_{pi} = E_{pf} + E_{kf}$$
$$mgh_i = mgh_f + \tfrac{1}{2}mv_f^2$$
$$(9.80)(45.0) = (9.80)(40.0) + \tfrac{1}{2}v_f^2$$
$$v_f = \boxed{9.90 m/s}$$

      Notice that the angle was not required for either part of the problem.

5. Tarzan is running at top speed (8.0 *m*/*s*) and grabs a vine hanging vertically from a tall tree in the jungle. How high can he swing upward?
   We assume that Tarzan is running along the ground and we will use the ground as our reference level.
   Initial: Tarzan has kinetic energy since he is moving, and no potential energy since he is at the reference level
   Final: Tarzan has potential energy since he is above the reference level, but no kinetic energy since he will come to rest at his highest point.

$$v_i = 8.0 m/s \quad v_f = 0 \quad h_i = 0 \quad h_f = ?$$

$$E_i = E_f$$
$$E_{ki} = E_{pf}$$
$$\tfrac{1}{2}\cancel{m}v_i^2 = \cancel{m}gh_f$$
$$\tfrac{1}{2}(8.0)^2 = (9.80)h_f$$
$$h_f = \boxed{3.3m}$$

6. In the high jump, with what minimum speed must an athlete leave the ground in order to lift his centre of mass 2.20 *m* and cross the bar with a speed of 0.80 *m*/*s*?

   We will use the ground as our reference level. The athlete has no initial potential energy since he is at the reference level.

$$h_i = 0 \quad h_f = 2.20m \quad v_i = ? \quad v_f = 0.80 m/s$$

$$E_i = E_f$$
$$E_{ki} = E_{pf} + E_{kf}$$
$$\tfrac{1}{2}\cancel{m}v_i^2 = \cancel{m}gh_f + \tfrac{1}{2}\cancel{m}v_f^2$$
$$\tfrac{1}{2}v_i^2 = (9.80)(2.20) + \tfrac{1}{2}(0.80)^2$$
$$v_i = \boxed{6.6 m/s}$$

7. A car starts off from rest on a hill 40.0 *m* high.
   a. How fast is it going at the bottom of the hill?

      Using the bottom of the hill as the reference level,

$$h_i = 40.0m \quad h_f = 0 \quad v_i = 0 \quad v_f = ?$$

$$E_i = E_f$$
$$E_{pi} = E_{kf}$$
$$\cancel{m}gh_i = \tfrac{1}{2}\cancel{m}v_f^2$$
$$(9.80)(40.0) = \tfrac{1}{2}v_f^2$$
$$v_f = \boxed{28.0 m/s}$$

   b. At what height will it have half its final speed?

$$h_i = 40.0 \quad h_f = ? \quad v_i = 0 \quad v_f = 14.0 m/s$$

$$E_i = E_f$$
$$E_{pi} = E_{pf} + E_{kf}$$
$$\cancel{m}gh_i = \cancel{m}gh_f + \tfrac{1}{2}\cancel{m}v_f^2$$
$$(9.80)(40.0) = (9.80)h_f + \tfrac{1}{2}(14.0)^2$$
$$h_f = \boxed{30.0m}$$

8. Balls A, B, and C are thrown from the top of a cliff with the same initial speed. Ball A is thrown vertically upward, ball B is thrown straight down, and ball C is thrown horizontally. Which ball has the greatest speed when it reaches the ground?

Again, this is a situation that is very complicated to analyze using kinematics equations but relatively easy to analyze using conservation of energy.

All of the balls have the same initial kinetic energy (same initial speed) and the same initial potential energy (same height). They must all have the same final energy. Since they all have the same final height, they must have the same final kinetic energy so that their total energy is the same; therefore, they must be travelling at the same speed. This does not mean that they will have the same velocity, as they will not all be moving in the same direction.