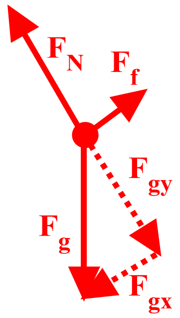
1.2.2 In Class or Homework Exercise

1. An 18.0 kg box is released on a 33.0^o incline and accelerates at 0.300 m/s^2. What is the coefficient of friction?



$m = 18.0kg$
$\theta = 33.0°$
$\mu = ?$
$\vec{a}_y = 0$
$\vec{a}_x = 0.300m/s^2$

First, we will find the components of the force of gravity:

$$F_{gx} = mg\sin\theta = (18.0)(9.80)\sin 33.0° = 96.1N$$

$$F_{gy} = mg\cos\theta = (18.0)(9.80)\cos 33.0° = 148N$$

Perpendicular Forces (using away from the ramp as positive)

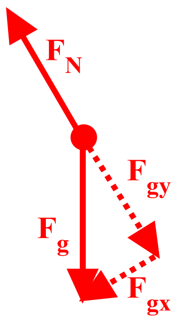
$$m\vec{a}_y = \sum \vec{F}_y$$
$$m\vec{a}_y = \vec{F}_N + \vec{F}_{gy}$$
$$m\vec{a}_y = F_N - F_{gy}$$
$$0 = F_N - 148$$
$$F_N = 148N$$

Parallel Forces (using down the ramp as positive)

$$m\vec{a}_x = \sum \vec{F}_x$$
$$m\vec{a}_x = \vec{F}_{gx} + \vec{F}_f$$
$$m\vec{a}_x = F_{gx} - F_f$$
$$(18.0)(0.300) = 96.1 - F_f$$
$$F_f = 90.7N$$

$$F_f = \mu F_N$$
$$90.7 = \mu(148)$$
$$= \boxed{0.613}$$

2. A box (mass is 455 g) is lying on a hill which has an inclination of 15.0^o with the horizontal.
   a. Ignoring friction, what is the acceleration of the box down the hill?



$$m = 455g = 0.455kg$$
$$\theta = 15.0°$$
$$\vec{a}_y = 0$$
$$\vec{a}_x = ?$$

First, we will find the components of the force of gravity:

$$F_{gx} = mg\sin\theta$$
$$= (0.455)(9.80)\sin 15.0°$$
$$= 1.15N$$

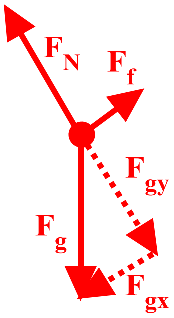
$$F_{gy} = mg\cos\theta$$
$$= (0.455)(9.80)\cos 15.0°$$
$$= 4.31N$$

Since there is no friction this time, we do not need to look at the perpendicular forces:

Parallel Forces (using down the ramp as positive)

$$m\vec{a}_x = \sum \vec{F}_x$$
$$m\vec{a}_x = \vec{F}_{gx}$$
$$m\vec{a}_x = F_{gx}$$
$$(0.455)\vec{a}_x = 1.15$$
$$\vec{a}_x = \boxed{2.53 m/s^2}$$

b. If there is a coefficient of friction of 0.20, will the box slide down the hill? If so, at what acceleration?



$$m = 455g = 0.455kg$$
$$F_{gx} = 1.15N$$
$$F_{gy} = 0.86N$$
$$\mu = 0.20$$
$$\vec{a}_y = 0$$
$$\vec{a}_x = ?$$

Since there is friction this time, we will need the normal force. We must therefore look at the perpendicular forces:

Perpendicular Forces

$$m\vec{a}_y = \sum \vec{F}_y$$
$$m\vec{a}_y = \vec{F}_N + \vec{F}_{gy}$$
$$m\vec{a}_y = F_N - F_{gy}$$
$$0 = F_N - 4.31$$
$$F_N = 4.31N$$

Parallel Forces (using down the ramp as positive)

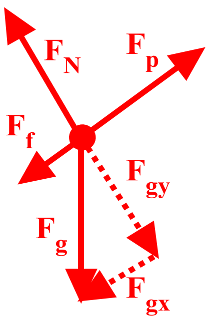
$$F_f = \mu F_N$$
$$= (0.20)(4.31)$$
$$= 0.86N$$

Since $F_{gx} > F_f$, the box will accelerate down the hill.

$$m\vec{a}_x = \sum \vec{F}_x$$
$$m\vec{a}_x = \vec{F}_{gx} + \vec{F}_f$$
$$m\vec{a}_x = F_{gx} - F_f$$
$$(0.455)\vec{a}_x = 1.15 - 0.86$$
$$\vec{a}_x = \boxed{0.64m/s^2}$$

c. How much force is required to push the box up the ramp at a constant speed?

Since the box is now going up the hill, friction must be down the hill:



$m = 0.455kg$

$F_f = 0.86N$

$F_{gx} = 1.15N$

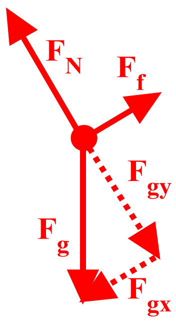
$\vec{a}_x = 0$

$\vec{a}_y = 0$

Using up the ramp as positive and looking at the parallel forces,

$$m\vec{a}_x = \sum \vec{F}_x$$
$$m\vec{a}_x = \vec{F}_{gx} + \vec{F}_f + \vec{F}_p$$
$$m\vec{a}_x = -F_{gx} - F_f + F_p$$
$$0 = F_p - F_{gx} - F_f$$
$$F_p = F_{gx} + F_f$$
$$= 1.15 + 0.86$$
$$= \boxed{2.01N}$$

3. A 165 kg piano is on a $25.0°$ ramp. The coefficient of friction is 0.30. Jack is responsible for seeing that nobody is killed by a runaway piano.
   a. How much force (and in what direction) must Jack exert so that the piano descends at a constant speed?
      Initially, we do not know what direction Jack has to apply his force so that the piano descends at a constant speed. If we look at the Free Body Diagram without Jack's force



$$m = 165kg$$
$$\theta = 25.0°$$
$$\mu = 0.30$$
$$\vec{a}_x = 0$$
$$\vec{a}_y = 0$$
$$F_p = ?$$

We see that we must compare F_{gx} and F_f

$$F_{gx} = mg\sin\theta = (165)(9.80)\sin 25.0° = 683N$$

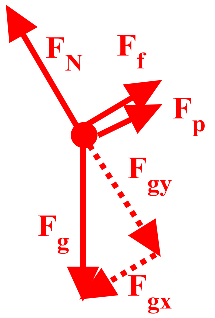
$$F_{gy} = mg\cos\theta = (165)(9.80)\cos 25.0° = 1470N$$

<u>Perpendicular Forces:</u>

$$m\vec{a}_y = \sum \vec{F}_y$$
$$m\vec{a}_y = \vec{F}_N + \vec{F}_{gy}$$
$$m\vec{a}_y = F_N - F_{gy}$$
$$0 = F_N - 1470$$
$$F_N = 1470N$$

$$F_f = \mu F_N$$
$$= (0.30)(1470)$$
$$= 440N$$

Since $F_{gx} > F_f$, the piano will accelerate down the ramp if Jack does nothing; therefore Jack must push up the ramp:



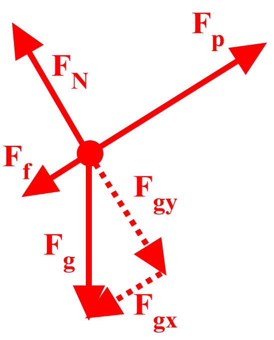
Using up the ramp as positive,

<u>Parallel Forces:</u>

$$m\vec{a}_x = \sum \vec{F}_x$$
$$m\vec{a}_x = \vec{F}_p + \vec{F}_f + \vec{F}_{gx}$$
$$0 = F_p + F_f - F_{gx}$$
$$F_p = F_{gx} - F_f$$
$$= 683 - 440$$
$$= \boxed{240N}$$

The force must be directed up the ramp.

b. How much force (and in what direction) must Jack exert so that the piano ascends at a constant speed?



$F_{gx} = 683N$

$F_f = 440N$

$\vec{a}_x = 0$

$F_p = ?$

Using up the ramp as positive,

$$m\vec{a}_x = \sum \vec{F}_x$$

$$m\vec{a}_x = \vec{F}_p + \vec{F}_f + \vec{F}_{gx}$$

$$0 = F_p - F_f - F_{gx}$$

$$F_p = F_{gx} + F_f$$

$$= 683 + 440$$

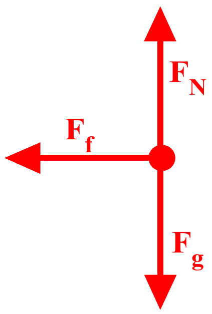
$$= \boxed{1120N}$$

The force must be directed up the ramp.

4. A car can decelerate at -5.5 m/s^2 when coming to rest on a level road. What would the deceleration be if the road inclines 15^o uphill?
The coefficient of friction will be the same in each situation (same surfaces). We must find this from the level surface part of the problem.
<u>Level Road</u>

$\vec{a} = -5.5m/s^2$

$\mu = ?$



Using the forward direction as positive,

$$m\vec{a} = \sum \vec{F}$$
$$m\vec{a} = -F_f$$
$$m\vec{a} = -\mu F_N$$
$$\cancel{m}\vec{a} = -\mu \cancel{m} g$$
$$-5.5 = -\mu(9.80)$$
$$\mu = 0.56$$

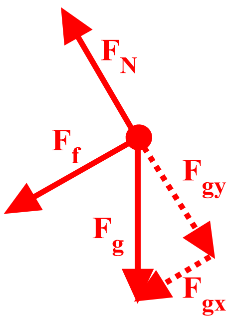
<u>Incline</u>

$\mu = 0.56$

$\theta = 15°$

$\vec{a}_y = 0$

$\vec{a}_x = ?$



Since there is no perpendicular acceleration, $F_N = F_{gy}$.

Looking at the parallel forces and using up the ramp as positive,

$$m\vec{a}_x = \sum \vec{F}_x$$
$$m\vec{a}_x = \vec{F}_{gx} + \vec{F}_f$$
$$m\vec{a}_x = -F_{gx} - F_f$$
$$\cancel{m}\vec{a}_x = -\cancel{m} g \sin\theta - \mu \cancel{m} g \cos\theta$$
$$\vec{a}_x = -(9.80)\sin 15° - (0.56)(9.80)\cos 15°$$
$$= \boxed{-7.8 m/s^2}$$

5. A sled is sliding down a hill and is 225 m from the bottom. It takes 13.5 s for the sled to reach the bottom. If the sled's speed at this location is 6.0 m/s and the slope of the hill is $30.0°$, what is the coefficient of friction between the hill and the sled?

   Using down the ramp as positive,

$\Delta\vec{d} = 225m$

$\vec{v}_i = 6.0m/s$

$t = 13.5s$

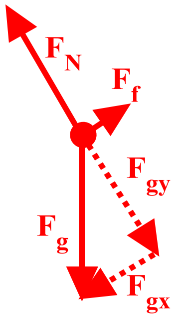
$\theta = 30.0°$

$\mu = ?$

$\Delta\vec{d}_x = \vec{v}_i t + \frac{1}{2}\vec{a}_x t^2$

$225 = (6.0)(13.5) + \frac{1}{2}\vec{a}_x (13.5)^2$

$\vec{a}_x = 1.6m/s^2$



$m\vec{a}_x = \sum \vec{F}_x$

$m\vec{a}_x = \vec{F}_{gx} + \vec{F}_f$

$m\vec{a}_x = F_{gx} - F_f$

$m\vec{a}_x = F_{gx} - \mu F_N$

$\not{m}\vec{a}_x = \not{m}g \sin\theta - \mu \not{m}g \cos\theta$

$1.6 = (9.80)\sin 30.0° - \mu(9.80)\cos 30.0°$

$\mu = \boxed{0.39}$

6. A 5.0 kg mass is on a ramp that is inclined at 30^o with the horizontal. A rope attached to the 5.0 kg block goes up the ramp and over a pulley, where it is attached to a 4.2 kg block that is hanging in mid air. The coefficient of friction between the 5.0 kg block and the ramp is 0.10. What is the acceleration of this system?

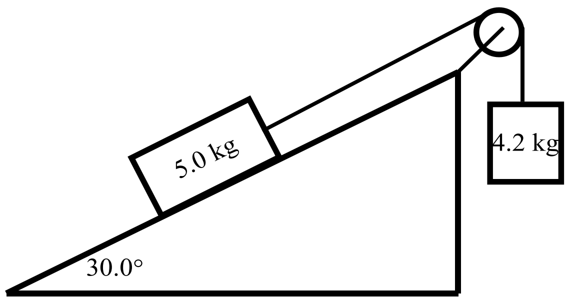


Figure 19: Diagram for Question 6

$m_1 = 5.0kg$
$m_2 = 4.2kg$
$\mu = 0.10$
$\vec{a} = ?$

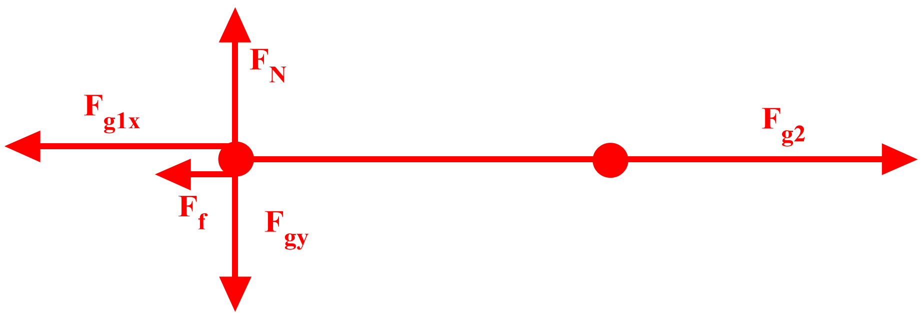
We must first determine the direction of acceleration of the system by comparing F_{g2} and F_{g1x}.

$$\begin{aligned} F_{g1x} &= m_1 g \sin\theta \\ &= (5.0)(9.80)\sin 30.0^\circ \\ &= 24.5N \end{aligned}$$

$$\begin{aligned} F_{g2} &= m_2 g \\ &= (4.2)(9.80) \\ &= 41.2N \end{aligned}$$

Since $F_{g2} > F_{g1x}$, the system will accelerate clockwise and friction on the 5.0 kg object will be down the ramp.

If we treat the whole system as one object (as in Unit 3), our linearized free body diagram will look like this:



The tension in the rope can be ignored since we are treating the system as one big object.

$$\begin{aligned} F_f &= \mu F_N \\ &= \mu F_{gy} \\ &= \mu mg\cos\theta \\ &= (0.10)(5.0)(9.80)\cos 30.0^\circ \\ &= 4.2N \end{aligned}$$

Using clockwise as the positive direction (toward the 4.2 kg mass),

$$m\vec{a} = \sum \vec{F}$$

$$m_t\vec{a} = \vec{F}_{g2} + \vec{F}_{g1x} + \vec{F}_f$$

$$m_t\vec{a} = F_{g2} - F_{g1x} - F_f$$

$$(9.2)\vec{a} = 41.2 - 24.5 - 4.2$$

$$\vec{a} = \boxed{1.4 m/s^2}$$

7. A force of 500.0 N applied to a rope held at 30.0^o above the surface of a ramp is required to pull a box weighing 1000.0 N at a constant velocity up the plane. The ramp has a base of 14.0 m and a length of 15.0 m. What is the coefficient of friction?

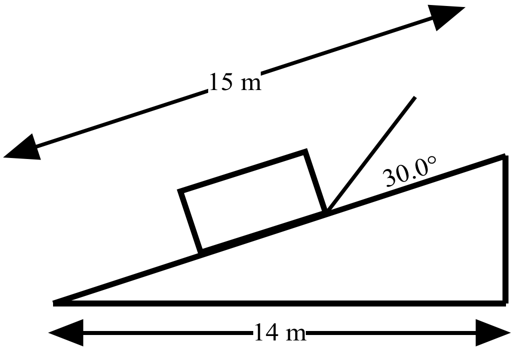
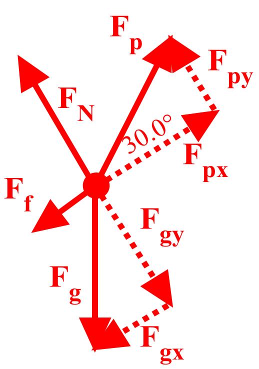


Figure 20: Diagram for Question 7

$$F_p = 500.0N$$
$$\theta = 30.0°$$
$$F_g = 1000.0N$$
$$\vec{a}_x = 0$$
$$\vec{a}_y = 0$$
$$\mu = ?$$

$$F_g = mg$$
$$1000.0 = m(9.80)$$
$$m = 102kg$$



We must break the applied force up into components that are parallel and perpendicular to the ramp.

$$F_{px} = F_p \cos\theta$$
$$= (500.0)(\cos 30.0°)$$
$$= 433N$$

$$F_{py} = F_p \sin\theta$$
$$= (500.0)(\sin 30.0°)$$
$$= 250.\,N$$

Next we must find the angle that the ramp makes with the horizontal so that we can find the components of the force of gravity:

$$\cos\theta = \frac{14}{15}$$
$$\theta = 21°$$

$$F_{gx} = mg\sin\theta$$
$$= (102)(9.80)\sin 21°$$
$$= 360N$$

$$F_{gy} = mg\cos\theta$$
$$= (102)(9.80)\cos 21°$$
$$= 930N$$

<u>Perpendicular Forces</u>

$$m\vec{a}_y = \sum \vec{F}_y$$
$$m\vec{a}_y = \vec{F}_{py} + \vec{F}_N + \vec{F}_{gy}$$
$$0 = F_{py} + F_N - F_{gy}$$
$$0 = 250 + F_N - 930$$
$$F_N = 680N$$

<u>Parallel Forces</u> (using up the ramp as positive)

$$m\vec{a}_x = \sum \vec{F}_x$$
$$m\vec{a}_x = \vec{F}_{px} + \vec{F}_{gx} + \vec{F}_f$$
$$0 = F_{px} - F_{gx} - F_f$$
$$0 = 433 - 360 - F_f$$
$$F_f = 73N$$

$$F_f = \mu F_N$$
$$73 = \mu(680)$$
$$\mu = \boxed{0.11}$$

8. If a bicyclist (75 kg) can coast down a 5.6^o hill at a steady speed of 7.0 km/h, how much force must be applied to climb the hill at the same speed?
We can assume the same magnitude for the force of friction on the way up and the way down the hill:
<u>Down the Hill</u>

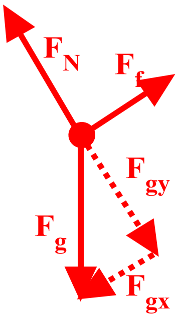
$m = 75kg$

$\theta = 5.6°$

$\vec{a} = 0$

$\vec{v} = 7.0km/h$

$F_f = ?$



Parallel Forces (Using down the ramp as positive)

$$m\vec{a}_x = \sum \vec{F}_x$$

$$m\vec{a}_x = \vec{F}_{gx} + \vec{F}_f$$

$$0 = F_{gx} - F_f$$

$$F_f = mg\sin\theta$$

$$= (75)(9.80)\sin(5.6°)$$

$$= 72N$$

Up the Hill

$m = 75kg$

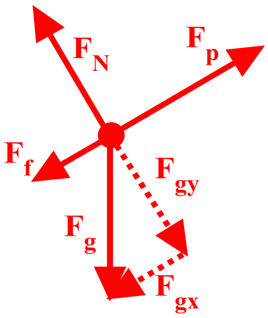
$\theta = 5.6°$

$\vec{a} = 0$

$\vec{v} = 7.0km/h$

$F_f = 72N$

$F_p = ?$



Parallel Forces (Using down the ramp as positive)

$$m\vec{a}_x = \sum \vec{F}_x$$

$$m\vec{a}_x = \vec{F}_{gx} + \vec{F}_f + \vec{F}_p$$

$$0 = F_{gx} + F_f - F_p$$

$$F_p = F_{gx} + F_f$$

$$= 72 + 72$$

$$= \boxed{144N}$$